Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.1-a+b-x+c-x^2-^p

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [113]. This is test number [19].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Fricas	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Mupad	100.00 (113)	0.00 (0)
Maxima	95.58 (108)	4.42 (5)
Giac	93.81 (106)	6.19 (7)
IntegrateAlgebraic	62.83 (71)	37.17 (42)
Sympy	41.59 (47)	% 58.41 (66)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

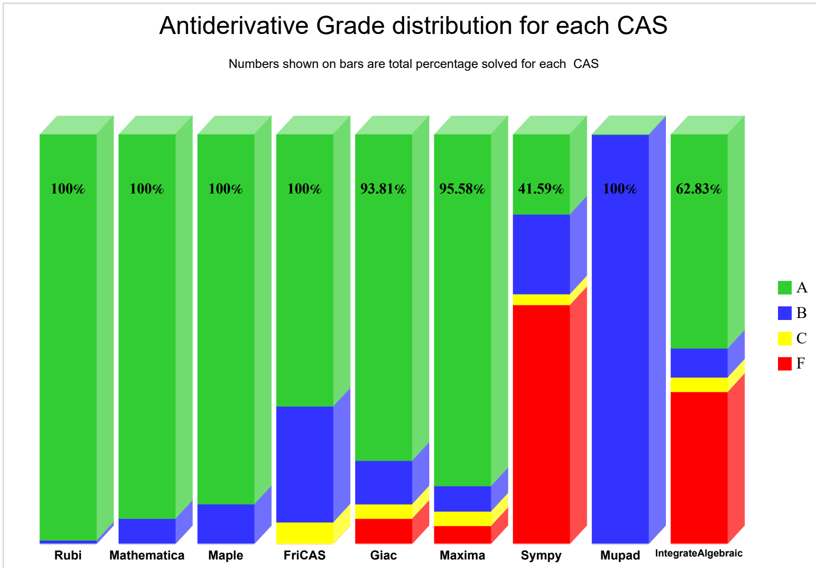
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

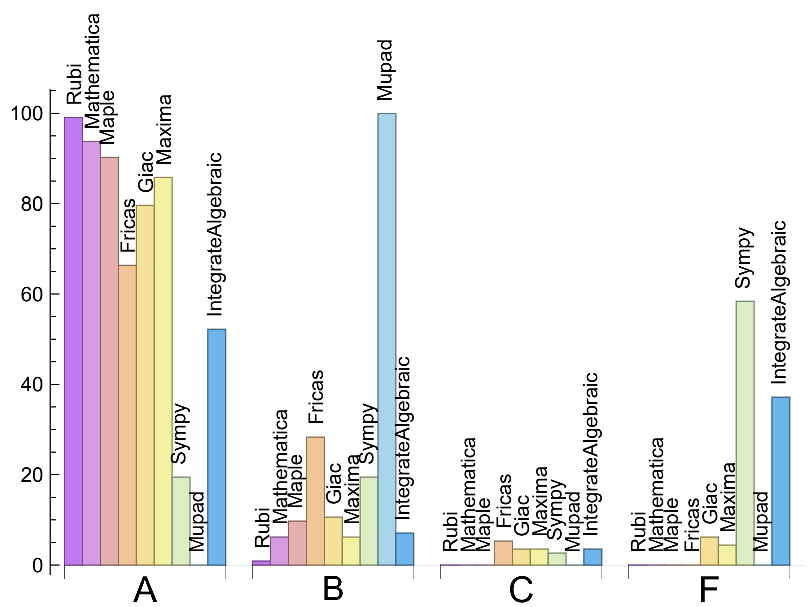
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.12	0.88	0.00	0.00
Mathematica	93.81	6.19	0.00	0.00
Maple	90.27	9.73	0.00	0.00
Maxima	85.84	6.19	3.54	4.42
Giac	79.65	10.62	3.54	6.19
Fricas	66.37	28.32	5.31	0.00
IntegrateAlgebraic	52.21	7.08	3.54	37.17
Sympy	19.47	19.47	2.65	58.41
Mupad	N/A	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
IntegrateAlgebraic	42	100.00 %	0.00 %	0.00 %
Giac	7	42.86 %	0.00 %	57.14 %
Maxima	5	0.00 %	0.00 %	100.00 %
Sympy	66	100.00 %	0.00 %	0.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

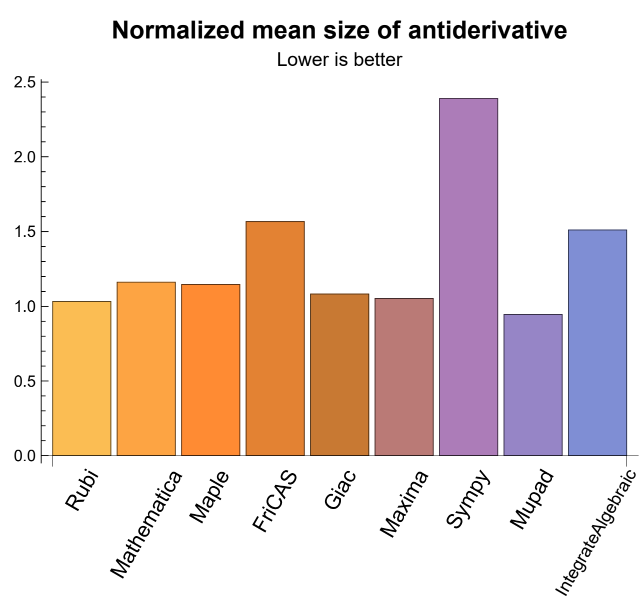
1.3 Performance

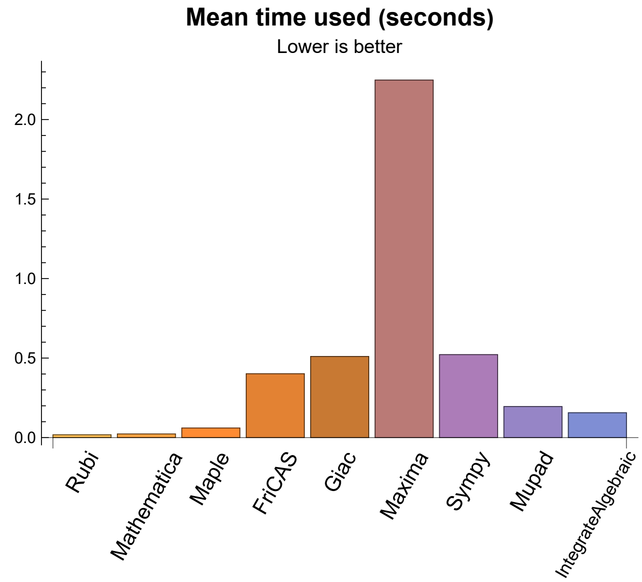
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	41.58	1.03	35.00	1.00
Mathematica	0.02	45.42	1.16	34.00	1.00
Maple	0.06	56.69	1.15	32.00	0.86
Maxima	2.25	45.49	1.05	33.50	1.03
Fricas	0.40	63.31	1.57	43.00	1.21
Sympy	0.52	118.68	2.39	70.00	1.81
Giac	0.51	45.38	1.08	33.00	0.89
Mupad	0.20	39.72	0.94	29.00	0.80
IntegrateAlgebraic	0.16	49.93	1.51	43.00	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{14, 15, 83\}$

IntegrateAlgebraic $\{48\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```python
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

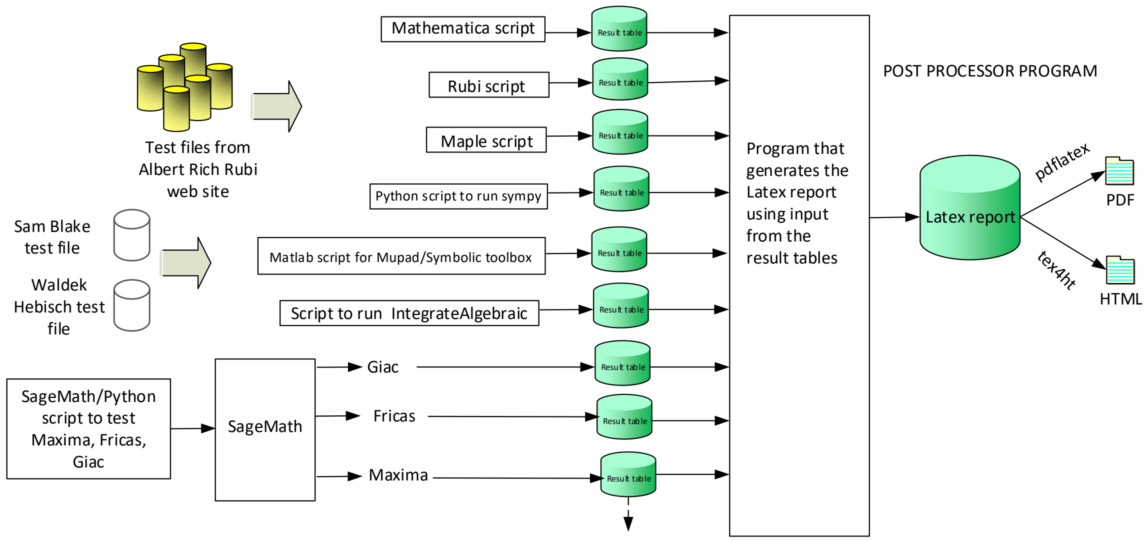
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

   The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

   The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form “{n,n,..}” which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 64 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 17, 21, 25, 28, 29, 64, 83 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 112, 113 }

B grade: { 25, 55, 56, 57, 58, 64, 82, 83, 106, 107, 108 }

C grade: { }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 59, 60, 61, 62, 63, 65, 66, 67, 68, 71, 72, 73, 74, 75, 78, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 17, 55, 56, 57, 58, 64, 82 }

C grade: { 52, 54, 92, 103 }

F grade: { 69, 70, 76, 77, 79 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 59, 60, 62, 63, 67, 69, 70, 72, 73, 74, 75, 79, 82, 83, 84, 86, 87, 88, 89, 90, 91, 93, 94, 97, 100, 102, 104, 111, 112, 113 }

B grade: { 17, 18, 21, 25, 27, 55, 56, 57, 58, 61, 64, 65, 66, 68, 71, 76, 77, 78, 80, 81, 85, 95, 96, 98, 99, 101, 105, 106, 107, 108, 109, 110 }

C grade: { 51, 52, 53, 54, 92, 103 }

F grade: { }

2.1.6 Sympy

A grade: { 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 49, 50, 59, 60, 61, 62, 63, 72, 73, 74, 75, 113 }

B grade: { 34, 35, 42, 43, 44, 55, 56, 57, 58, 64, 65, 66, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81 }

C grade: { 67, 82, 83 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 45, 46, 47, 48, 51, 52, 53, 54, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 49, 50, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 99, 100, 101, 102, 104, 105, 109, 110, 111, 112, 113 }

B grade: { 45, 55, 56, 57, 58, 64, 82, 95, 98, 106, 107, 108 }

C grade: { 51, 52, 53, 54 }

F grade: { 17, 18, 19, 20, 48, 92, 103 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

C grade: { }

F grade: { }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48, 50, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 97, 98, 100, 102, 104, 106, 109, 110, 111, 112 }

B grade: { 25, 26, 96, 99, 101, 105, 107, 108 }

C grade: { 52, 54, 92, 103 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 45, 46, 49, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 113 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	142	173	180	258	0	132	151	134
N.S.	1	1.00	0.97	1.18	1.22	1.76	0.00	0.90	1.03	0.91
time (sec)	N/A	0.054	0.223	0.046	1.341	0.419	0.000	0.632	0.735	0.527

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	119	91	130	69	0	1	100	92
N.S.	1	1.00	0.98	0.75	1.07	0.57	0.00	0.01	0.83	0.76
time (sec)	N/A	0.031	0.095	0.118	3.001	0.412	0.000	0.573	0.292	0.411

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	88	71	103	59	0	1	80	80
N.S.	1	1.00	0.93	0.75	1.08	0.62	0.00	0.01	0.84	0.84
time (sec)	N/A	0.022	0.079	0.093	3.012	0.417	0.000	0.460	0.298	0.271

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	76	51	76	49	0	1	60	68
N.S.	1	1.00	1.10	0.74	1.10	0.71	0.00	0.01	0.87	0.99
time (sec)	N/A	0.015	0.064	0.118	2.985	0.411	0.000	0.610	0.162	0.177

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	64	31	49	39	0	1	39	56
N.S.	1	1.00	1.49	0.72	1.14	0.91	0.00	0.02	0.91	1.30
time (sec)	N/A	0.010	0.049	0.105	2.963	0.408	0.000	0.625	0.087	0.103

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	88	82	117	68	0	57	81	78
N.S.	1	1.00	0.87	0.81	1.16	0.67	0.00	0.56	0.80	0.77
time (sec)	N/A	0.028	0.067	0.044	3.058	0.399	0.000	0.471	0.166	0.306

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	78	64	90	58	0	47	63	68
N.S.	1	1.00	0.99	0.81	1.14	0.73	0.00	0.59	0.80	0.86
time (sec)	N/A	0.019	0.047	0.046	2.944	0.390	0.000	0.454	0.242	0.221

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	68	46	63	48	0	37	45	58
N.S.	1	1.00	1.19	0.81	1.11	0.84	0.00	0.65	0.79	1.02
time (sec)	N/A	0.013	0.046	0.048	2.898	0.407	0.000	0.523	0.112	0.150

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	58	28	36	38	0	27	26	48
N.S.	1	1.00	1.66	0.80	1.03	1.09	0.00	0.77	0.74	1.37
time (sec)	N/A	0.009	0.032	0.051	2.885	0.417	0.000	0.477	0.052	0.095

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	32	28	36	35	0	25	26	41
N.S.	1	1.00	0.91	0.80	1.03	1.00	0.00	0.71	0.74	1.17
time (sec)	N/A	0.010	0.039	0.047	2.974	0.403	0.000	0.361	0.048	0.105

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	58	28	36	38	0	27	26	48
N.S.	1	1.00	1.66	0.80	1.03	1.09	0.00	0.77	0.74	1.37
time (sec)	N/A	0.010	0.034	0.045	2.926	0.409	0.000	0.435	0.054	0.106

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	42	55	43	0	35	39	51
N.S.	1	1.00	0.86	0.82	1.08	0.84	0.00	0.69	0.76	1.00
time (sec)	N/A	0.010	0.063	0.043	2.888	0.403	0.000	0.543	0.194	0.147

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	40	33	41	32	0	33	29	37
N.S.	1	1.00	1.14	0.94	1.17	0.91	0.00	0.94	0.83	1.06
time (sec)	N/A	0.007	0.027	0.046	1.418	0.411	0.000	0.520	0.199	0.103

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	37	37	48	33	43	32	0	33	29	39
N.S.	1	1.00	1.30	0.89	1.16	0.86	0.00	0.89	0.78	1.05
time (sec)	N/A	0.007	0.036	0.045	1.314	0.380	0.000	0.517	0.109	0.093

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	39	39	46	33	43	36	0	37	29	43
N.S.	1	1.00	1.18	0.85	1.10	0.92	0.00	0.95	0.74	1.10
time (sec)	N/A	0.007	0.026	0.048	1.349	0.398	0.000	0.415	0.195	0.094

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	70	75	111	105	0	74	96	82
N.S.	1	1.00	0.84	0.90	1.34	1.27	0.00	0.89	1.16	0.99
time (sec)	N/A	0.018	0.021	0.048	1.343	0.390	0.000	0.492	0.271	0.381

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	B	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	53	10	21	19	0	0	19	29
N.S.	1	1.00	3.31	0.62	1.31	1.19	0.00	0.00	1.19	1.81
time (sec)	N/A	0.006	0.012	0.105	2.932	0.413	0.000	0.000	0.276	0.103

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	24	21	28	39	0	0	20	38
N.S.	1	1.00	0.92	0.81	1.08	1.50	0.00	0.00	0.77	1.46
time (sec)	N/A	0.003	0.005	0.102	1.372	0.418	0.000	0.000	0.045	0.165

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	36	42	55	62	0	0	31	50
N.S.	1	1.00	0.68	0.79	1.04	1.17	0.00	0.00	0.58	0.94
time (sec)	N/A	0.007	0.010	0.099	1.368	0.410	0.000	0.000	0.124	0.222

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	48	62	82	82	0	0	40	62
N.S.	1	1.00	0.61	0.78	1.04	1.04	0.00	0.00	0.51	0.78
time (sec)	N/A	0.012	0.012	0.101	1.370	0.415	0.000	0.000	0.286	0.308

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	40	9	8	19	0	8	8	23
N.S.	1	1.00	3.33	0.75	0.67	1.58	0.00	0.67	0.67	1.92
time (sec)	N/A	0.006	0.012	0.043	2.953	0.393	0.000	0.498	0.106	0.096

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	25	28	29	0	29	18	32
N.S.	1	1.00	0.95	1.14	1.27	1.32	0.00	1.32	0.82	1.45
time (sec)	N/A	0.002	0.005	0.051	1.344	0.388	0.000	0.507	0.136	0.147

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	31	35	55	46	0	39	28	42
N.S.	1	1.00	0.69	0.78	1.22	1.02	0.00	0.87	0.62	0.93
time (sec)	N/A	0.006	0.009	0.044	1.363	0.406	0.000	0.408	0.032	0.181

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	51	45	82	61	0	49	73	52
N.S.	1	1.00	0.76	0.67	1.22	0.91	0.00	0.73	1.09	0.78
time (sec)	N/A	0.012	0.013	0.047	1.366	0.396	0.000	0.504	0.195	0.235

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	47	35	21	27	0	15	42	97
N.S.	1	1.00	3.92	2.92	1.75	2.25	0.00	1.25	3.50	8.08
time (sec)	N/A	0.008	0.013	0.072	2.916	0.407	0.000	0.520	0.312	0.183

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	45	37	29	27	0	36	36	117
N.S.	1	1.00	1.88	1.54	1.21	1.12	0.00	1.50	1.50	4.88
time (sec)	N/A	0.008	0.017	0.052	1.375	0.383	0.000	0.546	0.226	0.251

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	14	7	8	18	0	6	6	20
N.S.	1	1.00	1.40	0.70	0.80	1.80	0.00	0.60	0.60	2.00
time (sec)	N/A	0.006	0.008	0.046	3.034	0.416	0.000	0.392	0.109	0.092

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	33	14	17	17	0	18	11	19
N.S.	1	1.00	2.06	0.88	1.06	1.06	0.00	1.12	0.69	1.19
time (sec)	N/A	0.004	0.006	0.041	1.311	0.395	0.000	0.540	0.511	0.081

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	33	14	17	17	0	18	11	19
N.S.	1	1.00	2.06	0.88	1.06	1.06	0.00	1.12	0.69	1.19
time (sec)	N/A	0.004	0.012	0.051	1.327	0.385	0.000	0.539	0.529	0.080

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	44	43	43	49	43	43	0
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84	0.00
time (sec)	N/A	0.017	0.002	0.041	1.313	0.352	0.070	0.389	0.026	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	32	31	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89	0.00
time (sec)	N/A	0.011	0.002	0.040	1.348	0.352	0.070	0.529	0.039	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.007	0.001	0.038	1.364	0.353	0.063	0.432	0.029	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.002	0.000	0.042	1.293	0.359	0.062	0.352	0.017	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.005	0.009	0.040	2.902	0.406	0.141	0.395	0.066	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.010	0.024	0.047	3.112	0.416	0.223	0.417	0.143	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	51	58	188	105	45	55	0
N.S.	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89	0.00
time (sec)	N/A	0.016	0.033	0.049	2.987	0.393	0.339	0.339	0.162	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	76	66	58	146	97	63	37	71
N.S.	1	1.00	0.90	0.79	0.69	1.74	1.15	0.75	0.44	0.85
time (sec)	N/A	0.022	0.108	0.044	1.380	0.432	4.426	0.359	0.204	0.092

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	51	43	124	70	49	37	60
N.S.	1	1.00	1.00	0.78	0.66	1.91	1.08	0.75	0.57	0.92
time (sec)	N/A	0.014	0.080	0.044	1.303	0.409	3.008	0.427	0.157	0.067

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	49	36	28	94	41	37	35	48
N.S.	1	1.00	1.07	0.78	0.61	2.04	0.89	0.80	0.76	1.04
time (sec)	N/A	0.009	0.019	0.043	1.357	0.424	1.907	0.399	0.127	0.042

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	13	59	17	23	20	28
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80	1.12
time (sec)	N/A	0.005	0.006	0.042	1.390	0.408	1.051	0.406	0.187	0.025

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	23	17	14	14	16
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88	1.00
time (sec)	N/A	0.002	0.004	0.044	1.253	0.400	0.647	0.476	0.032	0.040

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	26	31	47	95	27	28	29
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72	0.74
time (sec)	N/A	0.006	0.008	0.045	1.020	0.386	0.862	0.511	0.192	0.060

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	37	46	69	413	41	44	40
N.S.	1	1.00	0.69	0.64	0.79	1.19	7.12	0.71	0.76	0.69
time (sec)	N/A	0.010	0.010	0.043	1.313	0.402	1.461	0.456	0.193	0.076

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	51	48	61	91	1265	55	61	51
N.S.	1	1.00	0.66	0.62	0.79	1.18	16.43	0.71	0.79	0.66
time (sec)	N/A	0.016	0.013	0.046	1.350	0.413	2.203	0.410	0.205	0.088

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	35	30	19	0	45	19	0
N.S.	1	1.00	0.87	1.52	1.30	0.83	0.00	1.96	0.83	0.00
time (sec)	N/A	0.003	0.037	0.046	2.957	0.389	0.000	0.393	0.049	0.141

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	25	25	30	9	0	26	19	0
N.S.	1	1.00	1.09	1.09	1.30	0.39	0.00	1.13	0.83	0.00
time (sec)	N/A	0.003	0.005	0.044	2.858	0.369	0.000	0.377	0.046	0.085

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	26	23	6	8	0	25	14	24
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.00	0.86	0.48	0.83
time (sec)	N/A	0.005	0.005	0.048	2.830	0.387	0.000	0.286	0.283	0.078

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	20	17	9	14	0	0	21	27
N.S.	1	1.00	0.80	0.68	0.36	0.56	0.00	0.00	0.84	1.08
time (sec)	N/A	0.003	0.005	0.046	2.892	0.390	0.000	0.000	0.170	0.175

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	25	25	30	9	8	26	13	0
N.S.	1	1.00	1.09	1.09	1.30	0.39	0.35	1.13	0.57	0.00
time (sec)	N/A	0.003	0.009	0.045	3.010	0.382	0.075	0.346	0.113	0.081

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	26	23	6	8	7	15	14	24
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.24	0.52	0.48	0.83
time (sec)	N/A	0.005	0.008	0.050	2.969	0.385	0.078	0.456	0.353	0.075

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	27	27	30	9	0	26	18	0
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78	0.00
time (sec)	N/A	0.003	0.007	0.047	3.015	0.383	0.000	0.375	0.327	0.084

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	F	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	28	25	6	6	0	23	15	30
N.S.	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52	1.03
time (sec)	N/A	0.005	0.006	0.049	2.898	0.385	0.000	0.447	0.245	0.086

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	27	27	30	9	0	26	18	0
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78	0.00
time (sec)	N/A	0.003	0.006	0.045	2.962	0.399	0.000	0.407	0.067	0.089

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	F	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	28	25	6	6	0	23	15	30
N.S.	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52	1.03
time (sec)	N/A	0.006	0.004	0.049	2.777	0.379	0.000	0.345	0.295	0.084

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	207	636	234	233	253	334	184	0
N.S.	1	1.00	1.90	5.83	2.15	2.14	2.32	3.06	1.69	0.00
time (sec)	N/A	0.140	0.030	0.039	1.413	0.382	0.194	0.429	0.314	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	207	636	234	235	250	334	184	0
N.S.	1	1.00	1.90	5.83	2.15	2.16	2.29	3.06	1.69	0.00
time (sec)	N/A	0.137	0.041	0.045	1.521	0.395	0.199	0.412	0.331	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	199	636	234	227	253	334	176	0
N.S.	1	1.00	1.83	5.83	2.15	2.08	2.32	3.06	1.61	0.00
time (sec)	N/A	0.142	0.028	0.039	1.409	0.392	0.202	0.450	0.337	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	207	636	234	235	248	334	184	0
N.S.	1	1.00	1.90	5.83	2.15	2.16	2.28	3.06	1.69	0.00
time (sec)	N/A	0.140	0.042	0.040	1.400	0.394	0.196	0.526	0.316	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	16	22	16	16	0
N.S.	1	1.00	1.00	0.94	0.89	0.89	1.22	0.89	0.89	0.00
time (sec)	N/A	0.011	0.009	0.042	2.955	0.389	0.114	0.369	0.139	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	7	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67	0.00
time (sec)	N/A	0.009	0.013	0.099	2.975	0.403	0.164	0.410	0.272	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	34	17	27	39	39	31	15	0
N.S.	1	1.00	1.79	0.89	1.42	2.05	2.05	1.63	0.79	0.00
time (sec)	N/A	0.019	0.019	0.043	2.953	0.375	0.119	0.458	0.209	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	10	15	8	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	0.62	0.00
time (sec)	N/A	0.005	0.003	0.049	1.327	0.391	0.103	0.507	0.082	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	15	15	14	17	8	0
N.S.	1	1.00	1.00	0.76	0.71	0.71	0.67	0.81	0.38	0.00
time (sec)	N/A	0.006	0.003	0.050	1.314	0.399	0.113	0.464	0.091	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	17	17	14	13	13	12	15	6	0
N.S.	1	2.83	2.83	2.33	2.17	2.17	2.00	2.50	1.00	0.00
time (sec)	N/A	0.004	0.003	0.048	1.288	0.396	0.104	0.330	0.176	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	38	50	76	40	23	0
N.S.	1	1.00	1.00	0.89	1.41	1.85	2.81	1.48	0.85	0.00
time (sec)	N/A	0.019	0.008	0.056	1.314	0.423	0.223	0.346	0.359	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	29	26	39	51	76	45	23	0
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85	0.00
time (sec)	N/A	0.019	0.008	0.050	1.326	0.418	0.233	0.383	0.386	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	28	27	41	87	27	23	0
N.S.	1	1.00	1.00	0.90	0.87	1.32	2.81	0.87	0.74	0.00
time (sec)	N/A	0.020	0.008	0.049	1.317	0.411	0.196	0.482	0.381	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	29	26	39	51	76	45	23	0
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85	0.00
time (sec)	N/A	0.018	0.007	0.051	1.441	0.399	0.234	0.532	0.418	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	35	0	113	124	34	46	0
N.S.	1	1.00	1.00	0.92	0.00	2.97	3.26	0.89	1.21	0.00
time (sec)	N/A	0.034	0.010	0.062	0.000	0.404	0.221	0.432	0.229	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	34	35	0	124	100	30	33	0
N.S.	1	1.00	0.97	1.00	0.00	3.54	2.86	0.86	0.94	0.00
time (sec)	N/A	0.027	0.009	0.064	0.000	0.407	0.232	0.365	0.273	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	41	31	49	67	102	55	28	0
N.S.	1	1.00	1.28	0.97	1.53	2.09	3.19	1.72	0.88	0.00
time (sec)	N/A	0.025	0.010	0.072	2.910	0.392	0.255	0.445	0.227	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	37	36	45	39	36	33	0
N.S.	1	1.00	1.00	0.86	0.84	1.05	0.91	0.84	0.77	0.00
time (sec)	N/A	0.015	0.022	0.037	2.274	0.391	0.150	0.580	0.038	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	62	37	47	68	58	51	34	0
N.S.	1	1.00	1.44	0.86	1.09	1.58	1.35	1.19	0.79	0.00
time (sec)	N/A	0.016	0.024	0.055	2.807	0.393	0.151	0.522	0.163	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	33	32	34	53	31	36	34	0
N.S.	1	1.00	0.97	0.94	1.00	1.56	0.91	1.06	1.00	0.00
time (sec)	N/A	0.008	0.010	0.053	1.316	0.383	0.140	0.454	0.200	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	32	34	53	32	36	34	0
N.S.	1	1.00	1.00	0.76	0.81	1.26	0.76	0.86	0.81	0.00
time (sec)	N/A	0.009	0.013	0.049	1.347	0.382	0.151	0.318	0.080	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	70	68	0	334	265	67	119	0
N.S.	1	1.00	0.99	0.96	0.00	4.70	3.73	0.94	1.68	0.00
time (sec)	N/A	0.040	0.058	0.056	0.000	0.403	0.589	0.404	0.171	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	86	0	317	230	75	107	0
N.S.	1	1.00	1.00	1.19	0.00	4.40	3.19	1.04	1.49	0.00
time (sec)	N/A	0.036	0.044	0.048	0.000	0.423	0.548	0.514	0.320	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	78	84	97	171	218	90	100	0
N.S.	1	1.00	1.13	1.22	1.41	2.48	3.16	1.30	1.45	0.00
time (sec)	N/A	0.034	0.060	0.052	2.839	0.405	0.603	0.526	0.316	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	65	111	0	89	212	100	110	0
N.S.	1	1.00	1.05	1.79	0.00	1.44	3.42	1.61	1.77	0.00
time (sec)	N/A	0.157	0.096	0.573	0.000	0.418	0.982	0.570	0.253	0.002

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	58	34	31	55	63	56	56	38	0
N.S.	1	1.76	1.03	0.94	1.67	1.91	1.70	1.70	1.15	0.00
time (sec)	N/A	0.036	0.032	0.080	1.389	0.414	0.292	0.527	0.258	0.002

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	58	32	31	51	59	56	54	38	0
N.S.	1	1.87	1.03	1.00	1.65	1.90	1.81	1.74	1.23	0.00
time (sec)	N/A	0.029	0.029	0.049	1.322	0.421	0.282	0.546	0.135	0.002

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	19	33	27	15	165	27	27	0
N.S.	1	1.00	1.12	1.94	1.59	0.88	9.71	1.59	1.59	0.00
time (sec)	N/A	0.018	0.020	0.241	2.954	0.418	0.192	0.466	0.117	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	23	23	56	39	33	21	70	33	42	0
N.S.	1	1.00	2.43	1.70	1.43	0.91	3.04	1.43	1.83	0.00
time (sec)	N/A	0.028	0.039	0.196	3.018	0.406	0.588	0.466	0.303	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	39	29	38	40	0	40	39	48
N.S.	1	1.00	1.03	0.76	1.00	1.05	0.00	1.05	1.03	1.26
time (sec)	N/A	0.011	0.016	0.055	3.042	0.383	0.000	0.468	0.080	0.073

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	38	53	0	24	23	49
N.S.	1	1.00	1.00	0.83	1.27	1.77	0.00	0.80	0.77	1.63
time (sec)	N/A	0.010	0.013	0.045	2.998	0.413	0.000	1.604	0.052	0.094

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	50	52	40	0	41	39	49
N.S.	1	1.00	1.00	1.02	1.06	0.82	0.00	0.84	0.80	1.00
time (sec)	N/A	0.010	0.015	0.053	2.869	0.382	0.000	0.429	0.210	0.107

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	46	35	46	58	0	53	48	59
N.S.	1	1.00	1.02	0.78	1.02	1.29	0.00	1.18	1.07	1.31
time (sec)	N/A	0.015	0.017	0.049	3.004	0.405	0.000	0.456	0.191	0.130

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	46	35	46	60	0	36	35	73
N.S.	1	1.00	1.02	0.78	1.02	1.33	0.00	0.80	0.78	1.62
time (sec)	N/A	0.016	0.017	0.046	2.972	0.392	0.000	0.470	0.052	0.252

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	50	58	58	0	54	48	59
N.S.	1	1.00	0.89	0.81	0.94	0.94	0.00	0.87	0.77	0.95
time (sec)	N/A	0.015	0.026	0.047	2.975	0.402	0.000	0.442	0.195	0.185

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	44	32	41	60	0	31	30	61
N.S.	1	1.00	1.02	0.74	0.95	1.40	0.00	0.72	0.70	1.42
time (sec)	N/A	0.012	0.019	0.045	3.083	0.408	0.000	0.401	0.150	0.135

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	53	50	58	58	0	54	48	66
N.S.	1	1.00	0.90	0.85	0.98	0.98	0.00	0.92	0.81	1.12
time (sec)	N/A	0.013	0.019	0.047	2.980	0.394	0.000	0.489	0.193	0.312

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	F	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	54	46	46	84	0	0	36	65
N.S.	1	1.00	0.92	0.78	0.78	1.42	0.00	0.00	0.61	1.10
time (sec)	N/A	0.016	0.025	0.050	2.894	0.404	0.000	0.000	0.054	0.148

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	50	58	58	0	54	48	59
N.S.	1	1.00	0.89	0.81	0.94	0.94	0.00	0.87	0.77	0.95
time (sec)	N/A	0.013	0.025	0.046	3.003	0.405	0.000	0.445	0.221	0.185

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	32	41	60	0	31	30	61
N.S.	1	1.00	1.03	0.82	1.05	1.54	0.00	0.79	0.77	1.56
time (sec)	N/A	0.008	0.018	0.043	3.078	0.409	0.000	0.526	0.052	0.131

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	9	8	20	0	20	20	24
N.S.	1	1.00	1.00	0.64	0.57	1.43	0.00	1.43	1.43	1.71
time (sec)	N/A	0.007	0.005	0.044	2.948	0.403	0.000	0.631	0.199	0.087

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	14	7	8	33	0	6	6	25
N.S.	1	1.00	1.40	0.70	0.80	3.30	0.00	0.60	0.60	2.50
time (sec)	N/A	0.007	0.006	0.046	3.011	0.405	0.000	0.518	0.132	0.106

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	30	22	20	0	21	20	24
N.S.	1	1.00	0.96	1.20	0.88	0.80	0.00	0.84	0.80	0.96
time (sec)	N/A	0.006	0.005	0.045	2.991	0.408	0.000	0.575	0.218	0.080

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	16	38	0	33	26	33
N.S.	1	1.00	1.00	0.83	0.89	2.11	0.00	1.83	1.44	1.83
time (sec)	N/A	0.010	0.006	0.049	2.877	0.410	0.000	0.616	0.225	0.093

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	15	16	40	0	16	16	39
N.S.	1	1.00	1.00	0.79	0.84	2.11	0.00	0.84	0.84	2.05
time (sec)	N/A	0.010	0.006	0.046	2.927	0.404	0.000	0.818	0.137	0.135

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	28	30	28	38	0	34	26	33
N.S.	1	1.00	0.80	0.86	0.80	1.09	0.00	0.97	0.74	0.94
time (sec)	N/A	0.008	0.005	0.041	3.010	0.417	0.000	0.802	0.236	0.118

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	11	40	0	11	11	35
N.S.	1	1.00	1.00	0.71	0.65	2.35	0.00	0.65	0.65	2.06
time (sec)	N/A	0.007	0.006	0.046	2.997	0.407	0.000	0.658	0.170	0.116

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	26	30	28	37	0	34	26	33
N.S.	1	1.00	0.81	0.94	0.88	1.16	0.00	1.06	0.81	1.03
time (sec)	N/A	0.008	0.006	0.043	2.987	0.410	0.000	0.562	0.228	0.089

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	F	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	28	26	16	65	0	0	17	38
N.S.	1	1.00	0.85	0.79	0.48	1.97	0.00	0.00	0.52	1.15
time (sec)	N/A	0.008	0.008	0.039	2.910	0.413	0.000	0.000	0.136	0.123

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	28	30	28	38	0	34	26	33
N.S.	1	1.00	0.80	0.86	0.80	1.09	0.00	0.97	0.74	0.94
time (sec)	N/A	0.008	0.006	0.043	2.927	0.405	0.000	0.731	0.226	0.114

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	40	0	11	11	35
N.S.	1	1.00	1.00	0.92	0.85	3.08	0.00	0.85	0.85	2.69
time (sec)	N/A	0.004	0.006	0.045	3.073	0.414	0.000	0.638	0.164	0.112

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	51	16	137	0	46	40	44
N.S.	1	1.00	1.00	2.32	0.73	6.23	0.00	2.09	1.82	2.00
time (sec)	N/A	0.013	0.026	0.076	1.402	0.438	0.000	0.754	0.423	0.263

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	44	19	141	0	53	46	128
N.S.	1	1.00	1.00	1.91	0.83	6.13	0.00	2.30	2.00	5.57
time (sec)	N/A	0.013	0.026	0.088	3.084	0.458	0.000	1.085	0.406	0.342

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	44	19	143	0	53	44	129
N.S.	1	1.00	1.00	2.20	0.95	7.15	0.00	2.65	2.20	6.45
time (sec)	N/A	0.011	0.025	0.079	2.890	0.475	0.000	0.817	0.400	0.331

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	24	26	38	0	17	15	29
N.S.	1	1.00	1.00	1.26	1.37	2.00	0.00	0.89	0.79	1.53
time (sec)	N/A	0.002	0.005	0.047	1.340	0.399	0.000	0.828	0.053	0.176

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	28	30	41	0	17	17	37
N.S.	1	1.00	1.00	1.22	1.30	1.78	0.00	0.74	0.74	1.61
time (sec)	N/A	0.003	0.006	0.046	1.236	0.397	0.000	0.774	0.057	0.197

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	26	30	29	0	29	19	33
N.S.	1	1.00	1.00	1.13	1.30	1.26	0.00	1.26	0.83	1.43
time (sec)	N/A	0.005	0.052	0.048	1.342	0.397	0.000	0.615	0.055	0.167

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	31	36	59	49	0	36	29	43
N.S.	1	1.00	0.72	0.84	1.37	1.14	0.00	0.84	0.67	1.00
time (sec)	N/A	0.007	0.009	0.044	1.411	0.426	0.000	0.777	0.035	0.279

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	17	17	16	19	20	16	32	0
N.S.	1	1.00	0.94	0.94	0.89	1.06	1.11	0.89	1.78	0.00
time (sec)	N/A	0.002	0.007	0.043	1.246	0.412	0.063	0.624	0.390	0.014

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [12] had the largest ratio of [.2727]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	13	0.231
2	A	6	3	1.00	15	0.200
3	A	5	3	1.00	15	0.200
4	A	4	3	1.00	15	0.200
5	A	3	3	1.00	15	0.200
6	A	6	3	1.00	13	0.231
7	A	5	3	1.00	13	0.231
8	A	4	3	1.00	13	0.231
9	A	3	3	1.00	13	0.231
10	A	3	3	1.00	13	0.231
11	A	3	3	1.00	13	0.231
12	A	4	3	1.00	11	0.273
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	11	0.273
15	A	3	3	1.00	11	0.273
16	A	3	2	1.00	13	0.154
17	A	2	2	1.00	15	0.133
18	A	1	1	1.00	15	0.067
19	A	2	2	1.00	15	0.133
20	A	3	2	1.00	15	0.133
21	A	2	2	1.00	13	0.154
22	A	1	1	1.00	13	0.077
23	A	2	2	1.00	13	0.154
24	A	3	2	1.00	13	0.154
25	A	2	2	1.00	16	0.125
26	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	2	2	1.00	13	0.154
28	A	2	2	1.00	11	0.182
29	A	2	2	1.00	11	0.182
30	A	2	1	1.00	9	0.111
31	A	2	1	1.00	9	0.111
32	A	2	1	1.00	9	0.111
33	A	1	0	1.00	7	0.000
34	A	1	1	1.00	9	0.111
35	A	2	2	1.00	9	0.222
36	A	3	2	1.00	9	0.222
37	A	5	3	1.00	11	0.273
38	A	4	3	1.00	11	0.273
39	A	3	3	1.00	11	0.273
40	A	2	2	1.00	11	0.182
41	A	1	1	1.00	11	0.091
42	A	2	2	1.00	11	0.182
43	A	3	2	1.00	11	0.182
44	A	4	2	1.00	11	0.182
45	A	1	1	1.00	14	0.071
46	A	1	1	1.00	14	0.071
47	A	2	2	1.00	14	0.143
48	A	1	1	1.00	14	0.071
49	A	1	1	1.00	14	0.071
50	A	2	2	1.00	14	0.143
51	A	1	1	1.00	14	0.071
52	A	2	2	1.00	14	0.143
53	A	1	1	1.00	14	0.071
54	A	2	2	1.00	14	0.143
55	A	3	2	1.00	23	0.087
56	A	3	2	1.00	23	0.087
57	A	3	2	1.00	23	0.087
58	A	3	2	1.00	23	0.087
59	A	2	2	1.00	12	0.167
60	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
61	A	2	2	1.00	12	0.167
62	A	3	2	1.00	12	0.167
63	A	3	2	1.00	12	0.167
64	B	3	2	2.83	10	0.200
65	A	2	2	1.00	12	0.167
66	A	2	2	1.00	12	0.167
67	A	2	2	1.00	12	0.167
68	A	2	2	1.00	12	0.167
69	A	2	2	1.00	12	0.167
70	A	2	2	1.00	13	0.154
71	A	2	2	1.00	14	0.143
72	A	3	3	1.00	12	0.250
73	A	3	3	1.00	12	0.250
74	A	4	3	1.00	12	0.250
75	A	4	3	1.00	12	0.250
76	A	3	3	1.00	12	0.250
77	A	3	3	1.00	13	0.231
78	A	3	3	1.00	14	0.214
79	A	2	2	1.00	40	0.050
80	A	3	2	1.76	30	0.067
81	A	3	2	1.87	31	0.065
82	A	2	2	1.00	14	0.143
83	A	2	2	1.00	16	0.125
84	A	3	3	1.00	14	0.214
85	A	3	3	1.00	14	0.214
86	A	3	3	1.00	14	0.214
87	A	3	3	1.00	14	0.214
88	A	3	3	1.00	14	0.214
89	A	3	3	1.00	14	0.214
90	A	3	3	1.00	14	0.214
91	A	3	3	1.00	14	0.214
92	A	3	3	1.00	14	0.214
93	A	3	3	1.00	14	0.214
94	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	14	0.143
96	A	2	2	1.00	14	0.143
97	A	2	2	1.00	14	0.143
98	A	2	2	1.00	14	0.143
99	A	2	2	1.00	14	0.143
100	A	2	2	1.00	14	0.143
101	A	2	2	1.00	14	0.143
102	A	2	2	1.00	14	0.143
103	A	2	2	1.00	14	0.143
104	A	2	2	1.00	14	0.143
105	A	2	2	1.00	14	0.143
106	A	2	2	1.00	27	0.074
107	A	2	2	1.00	30	0.067
108	A	2	2	1.00	28	0.071
109	A	1	1	1.00	12	0.083
110	A	1	1	1.00	14	0.071
111	A	1	1	1.00	16	0.062
112	A	2	2	1.00	14	0.143
113	A	1	1	1.00	7	0.143

Chapter 3

Listing of integrals

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Optimal. Leaf size=147

$$\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{(b+$$

Rubi [A] time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 620, 206}

$$-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b + 2*c*x)*(
b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(3
84*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) + (35*b^8*ArcTanh[(Sqrt[
c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{7/2}\,dx = \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} - \frac{\left(7b^2\right)\int \left(bx+cx^2\right)^{5/2}\,dx}{32c}$$

$$= -\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} + \frac{\left(35b^4\right)\int \left(bx+cx^2\right)^{3/2}\,dx}{768c^2}$$

$$= \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} - \frac{(35b\cdots}{\cdots}$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \cdots$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \cdots$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \cdots$$

Mathematica [A] time = 0.22, size = 142, normalized size = 0.97

$$\frac{\sqrt{x(b+cx)}\left(\frac{105b^{15/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}+\sqrt{c}\left(-105b^7+70b^6cx-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+25856b^2c^5x^5+21504bc^6x^6+6144c^7x^7\right)\right)}{49152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^7 + 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b^
4*c^3*x^3 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 6144*
c^7*x^7) + (105*b^(15/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[
1 + (c*x)/b])))/(49152*c^(9/2))
```

IntegrateAlgebraic [A] time = 0.53, size = 134, normalized size = 0.91

$$\frac{\sqrt{bx+cx^2}\left(-105b^7+70b^6cx-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+25856b^2c^5x^5+21504bc^6x^6+6144c^7x^7\right)}{49152c^4}-\frac{35b^8\log\left(-2\sqrt{c}\sqrt{bx+cx^2}+b+2cx\right)}{32768c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b*x + c*x^2]*(-105*b^7 + 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b^4*c^3*x^3
 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 6144*c^7*x^7))
```

```
/(49152*c^4) - (35*b^8*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[b*x + c*x^2]])/(32768
*c^(9/2))
```

fricas [A] time = 0.42, size = 258, normalized size = 1.76

$$\left[\frac{105\, b^{8} \sqrt{c} \log\left(2\, c x + b + 2\, \sqrt{c x^{2} + b x} \sqrt{c}\right) + 2\, \left(6144\, c^{8} x^{7} + 21504\, b c^{7} x^{6} + 25856\, b^{2} c^{6} x^{5} + 10880\, b^{3} c^{5} x^{4} + 48\, b^{4} c^{4} x^{3} - 56\, b^{5} c^{3} x^{2} + 70\, b^{6} c^{2} x - 105\, b^{7} c\right)\sqrt{c x^{2} + b x}}{98304\, c^{5}}, -\frac{105\, b^{8} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{2} + b x} \sqrt{-c}}{c x}\right) - \left(6144\, c^{8} x^{7} + 21504\, b c^{7} x^{6} + 25856\, b^{2} c^{6} x^{5} + 10880\, b^{3} c^{5} x^{4} + 48\, b^{4} c^{4} x^{3} - 56\, b^{5} c^{3} x^{2} + 70\, b^{6} c^{2} x - 105\, b^{7} c\right)\sqrt{c x^{2} + b x}}{49152\, c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/98304*(105*b^8*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*
(6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 + 4
8*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 + b*x
))/c^5, -1/49152*(105*b^8*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x))
 - (6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4
+ 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 +
b*x))/c^5]
```

giac [A] time = 0.63, size = 132, normalized size = 0.90

$$-\frac{35\, b^{8} \log\left({\left| -2\, {\left(\sqrt{c} x - \sqrt{c x^{2} + b x}\right)} \sqrt{c} - b \right|}\right)}{32768\, c^{\frac{9}{2}}} - \frac{1}{49152} \, {\left(\frac{105\, b^{7}}{c^{4}} - 2\, {\left(\frac{35\, b^{6}}{c^{3}} - 4\, {\left(\frac{7\, b^{5}}{c^{2}} - 2\, {\left(\frac{3\, b^{4}}{c} + 8\, {\left(85\, b^{3} + 2\, {\left(101\, b^{2} c + 12\, {\left(2\, c^{3} x + 7\, b c^{2}\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} \sqrt{c x^{2} + b x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -35/32768*b^8*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9
/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^2 - 2*(3*b^4/c + 8*
(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2)*x)*x)*x)*x)*x)*x)*sqrt(c*x^
2 + b*x)
```

maple [A] time = 0.05, size = 173, normalized size = 1.18

$$\frac{35 b^{8} \ln\left(\frac{c x+\frac{b}{2}}{\sqrt{c}}+\sqrt{c \,x^{2}+b x}\right)}{32768 c^{\frac{9}{2}}}-\frac{35 \sqrt{c \,x^{2}+b x}\, b^{6} x}{8192 c^{3}}-\frac{35 \sqrt{c \,x^{2}+b x}\, b^{7}}{16384 c^{4}}+\frac{35 \left(c \,x^{2}+b x \right)^{\frac{3}{2}} b^{4} x}{3072 c^{2}}+\frac{35 \left(c \,x^{2}+b x \right)^{\frac{3}{2}} b^{5}}{6144 c^{3}}-\frac{7 \left(c \,x^{2}+b x \right)^{\frac{5}{2}} b^{2} x}{192 c}-\frac{7 \left(c \,x^{2}+b x \right)^{\frac{5}{2}} b^{3}}{384 c^{2}}+\frac{\left(2 c x +b \right) \left(c \,x^{2}+b x \right)^{\frac{7}{2}}}{16 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(7/2),x)
```

```
[Out] 1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c-7/192*b^2/c*(c*x^2+b*x)^(5/2)*x-7/384*b^
3/c^2*(c*x^2+b*x)^(5/2)+35/3072*b^4/c^2*(c*x^2+b*x)^(3/2)*x+35/6144*b^5/c^3
*(c*x^2+b*x)^(3/2)-35/8192*b^6/c^3*(c*x^2+b*x)^(1/2)*x-35/16384*b^7/c^4*(c*
x^2+b*x)^(1/2)+35/32768*b^8/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2
))
```

maxima [A] time = 1.34, size = 180, normalized size = 1.22

$$\frac{1}{8}\left(cx^2+bx\right)^{\frac{7}{2}}x-\frac{35\sqrt{cx^2+bx}\,b^6x}{8192\,c^3}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^4x}{3072\,c^2}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^2x}{192\,c}+\frac{35\,b^8\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\,\sqrt{c}\right)}{32768\,c^{\frac{9}{2}}}-\frac{35\sqrt{cx^2+bx}\,b^7}{16384\,c^4}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^5}{6144\,c^3}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^3}{384\,c^2}+\frac{\left(cx^2+bx\right)^{\frac{7}{2}}b}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + b*x)^(7/2)*x - 35/8192*sqrt(c*x^2 + b*x)*b^6*x/c^3 + 35/3072*(
c*x^2 + b*x)^(3/2)*b^4*x/c^2 - 7/192*(c*x^2 + b*x)^(5/2)*b^2*x/c + 35/32768
*b^8*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) - 35/16384*sqrt(c
*x^2 + b*x)*b^7/c^4 + 35/6144*(c*x^2 + b*x)^(3/2)*b^5/c^3 - 7/384*(c*x^2 +
b*x)^(5/2)*b^3/c^2 + 1/16*(c*x^2 + b*x)^(7/2)*b/c
```

mupad [B] time = 0.74, size = 151, normalized size = 1.03

$$\frac{\left(c\,x^2+b\,x\right)^{7/2}\left(\frac{b}{2}+c\,x\right)}{8\,c}-\frac{7\,b^2\left(\frac{\left(c\,x^2+b\,x\right)^{5/2}\left(\frac{b}{2}+c\,x\right)}{6\,c}-\frac{5\,b^2\left(\frac{\left(c\,x^2+b\,x\right)^{3/2}\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{24\,c}\right)}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*x + c*x^2)^(7/2)*(b/2 + c*x))/(8*c) - (7*b^2*(((b*x + c*x^2)^(5/2)*(b/2
 + c*x))/(6*c) - (5*b^2*(((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2*(
(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x +
 c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)))/(32*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx+cx^2\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(7/2), x)
```

3.2 $\int \left(3ix + 4x^2\right)^{7/2} \, dx$

Optimal. Leaf size=121

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}$$

Rubi [A] time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {612, 619, 215}

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}+\frac{229635i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{16777216}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/2048 +
 ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + ((229635*I)/16777216)*ArcSin[1
- ((8*I)/3)*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{7/2} dx = \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{63}{128}\int \left(3ix+4x^2\right)^{5/2} dx$$

$$= \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} + \frac{945\int \left(3ix+4x^2\right)^{3/2} dx}{4096}$$

$$= \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} + \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2} +$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} +$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} +$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304} + \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072} + \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048} +$$

Mathematica [A] time = 0.09, size = 119, normalized size = 0.98

$$\frac{\sqrt{x(4x+3i)}\left(2\sqrt{3-4ix}\sqrt{x}\left(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76545i\right)-229635\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)\right)}{8388608\sqrt{3-4ix}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(2*Sqrt[3 - (4*I)*x]*Sqrt[x]*(76545*I - 68040*x - (725
76*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x^5 + (88080384*I)*x^6
+ 33554432*x^7) - 229635*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]]))/(83
88608*Sqrt[3 - (4*I)*x]*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.41, size = 92, normalized size = 0.76

$$\frac{\sqrt{4x^2+3ix}\left(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76545i\right)}{4194304} - \frac{229635\log\left(4\sqrt{4x^2+3ix}-8x-3i\right)}{16777216}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[(3*I)*x + 4*x^2]*(76545*I - 68040*x - (72576*I)*x^2 + 82944*x^3 - (25
067520*I)*x^4 - 79429632*x^5 + (88080384*I)*x^6 + 33554432*x^7))/4194304 -
(229635*Log[-3*I - 8*x + 4*Sqrt[(3*I)*x + 4*x^2]])/16777216
```

fricas [A] time = 0.41, size = 69, normalized size = 0.57

$$\frac{1}{268435456}\left(2147483648\,x^{7}+5637144576i\,x^{6}-5083496448\,x^{5}-1604321280i\,x^{4}+5308416\,x^{3}-4644864i\,x^{2}-4354560\,x+4898880i\right)\sqrt{4\,x^{2}+3i\,x}-\frac{229635}{16777216}\,\log\left(-2\,x+\sqrt{4\,x^{2}+3i\,x}-\frac{3}{4}i\right)-\frac{1165671}{268435456}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/268435456*(2147483648*x^7 + 5637144576*I*x^6 - 5083496448*x^5 - 160432128
0*I*x^4 + 5308416*x^3 - 4644864*I*x^2 - 4354560*x + 4898880*I)*sqrt(4*x^2 +
 3*I*x) - 229635/16777216*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 1165671
/268435456
```

giac [A] time = 0.57, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] 0
```

maple [A] time = 0.12, size = 91, normalized size = 0.75

$$\frac{229635\,\mathrm{arcsinh}\left(\frac{8x}{3}+i\right)}{16777216}+\frac{(8x+3i)\left(4x^{2}+3ix\right)^{\frac{7}{2}}}{64}+\frac{21\,(8x+3i)\left(4x^{2}+3ix\right)^{\frac{5}{2}}}{2048}+\frac{945\,(8x+3i)\left(4x^{2}+3ix\right)^{\frac{3}{2}}}{131072}+\frac{25515\,(8x+3i)\,\sqrt{4x^{2}+3ix}}{4194304}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(7/2),x)
```

```
[Out] 1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+94
5/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2
)^(1/2)+229635/16777216*arcsinh(8/3*x+I)
```

maxima [A] time = 3.00, size = 130, normalized size = 1.07

$$\frac{1}{8}\left(4\,x^{2}+3i\,x\right)^{\frac{7}{2}}x+\frac{3}{64}i\left(4\,x^{2}+3i\,x\right)^{\frac{7}{2}}+\frac{21}{256}\left(4\,x^{2}+3i\,x\right)^{\frac{5}{2}}x+\frac{63}{2048}i\left(4\,x^{2}+3i\,x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}x+\frac{2835}{131072}i\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{4\,x^{2}+3i\,x}\,x+\frac{76545}{4194304}i\sqrt{4\,x^{2}+3i\,x}+\frac{229635}{16777216}\log\left(8\,x+4\sqrt{4\,x^{2}+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/256*(4*x^2
+ 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 945/16384*(4*x^2 + 3*I
*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(3/2) + 25515/524288*sqrt(4*x^2
```

```
+ 3*I*x)*x + 76545/4194304*I*sqrt(4*x^2 + 3*I*x) + 229635/16777216*log(8*x
+ 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

mupad [B] time = 0.29, size = 100, normalized size = 0.83

$$\frac{229635\,\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{16777216}+\frac{945\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}{65536}+\frac{21\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{5/2}}{1024}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{7/2}}{32}+\frac{25515\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(7/2),x)
```

```
[Out] (229635*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/16777216 + (945*(4*x + 3i/2
)*(x*3i + 4*x^2)^(3/2))/65536 + (21*(4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/1024
 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(7/2))/32 + (25515*(x/2 + 3i/16)*(x*3i + 4*
x^2)^(1/2))/262144
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4x^2 + 3ix\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(7/2), x)
```

3.3 $\int \left(3ix+4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

Rubi [A] time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {612, 619, 215}

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*((3*I)*x +
4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + ((3645*I)/1
31072)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{5/2}\,dx = \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{15}{32}\int \left(3ix+4x^2\right)^{3/2}\,dx$$

$$= \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{405\int \sqrt{3ix+4x^2}\,dx}{2048}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{364}{}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{1215}{}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768} + \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024} + \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2} + \frac{364}{}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.93

$$\frac{\sqrt{x(4x+3i)}\left(524288x^5+983040ix^4-497664x^3-6912ix^2-6480x-\frac{10935\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}}+7290i\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (983040
*I)*x^4 + 524288*x^5 - (10935*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])
/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/196608
```

IntegrateAlgebraic [A] time = 0.27, size = 80, normalized size = 0.84

$$\frac{\sqrt{4x^2+3ix}\left(262144x^5+491520ix^4-248832x^3-3456ix^2-3240x+3645i\right)}{98304} - \frac{3645\log\left(4\sqrt{4x^2+3ix}-8x-3i\right)}{131072}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[(3*I)*x + 4*x^2]*(3645*I - 3240*x - (3456*I)*x^2 - 248832*x^3 + (4915
20*I)*x^4 + 262144*x^5))/98304 - (3645*Log[-3*I - 8*x + 4*Sqrt[(3*I)*x + 4*
x^2]])/131072
```

fricas [A] time = 0.42, size = 59, normalized size = 0.62

$$\frac{1}{3145728}\left(8388608\,x^5+15728640i\,x^4-7962624\,x^3-110592i\,x^2-103680\,x+116640i\right)\sqrt{4\,x^2+3i\,x}-\frac{3645}{131072}\,\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{8991}{1048576}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3145728*(8388608*x^5 + 15728640*I*x^4 - 7962624*x^3 - 110592*I*x^2 - 1036
80*x + 116640*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 +
3*I*x) - 3/4*I) - 8991/1048576
```

giac [A] time = 0.46, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 0
```

maple [A] time = 0.09, size = 71, normalized size = 0.75

$$\frac{3645\,\mathrm{arcsinh}\left(\frac{8x}{3}+i\right)}{131072}+\frac{(8x+3i)\left(4x^2+3ix\right)^{\frac{5}{2}}}{48}+\frac{15\,(8x+3i)\left(4x^2+3ix\right)^{\frac{3}{2}}}{1024}+\frac{405\,(8x+3i)\,\sqrt{4x^2+3ix}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*I*x)^(5/2),x)
```

```
[Out] 1/48*(8*x+3*I)*(4*x^2+3*I*x)^(5/2)+15/1024*(8*x+3*I)*(4*x^2+3*I*x)^(3/2)+40
5/32768*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+3645/131072*arcsinh(8/3*x+I)
```

maxima [A] time = 3.01, size = 103, normalized size = 1.08

$$\frac{1}{6}\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}x+\frac{1}{16}i\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}+\frac{15}{128}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{45}{1024}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{4\,x^2+3i\,x}\,x+\frac{1215}{32768}i\sqrt{4\,x^2+3i\,x}+\frac{3645}{131072}\,\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2
+ 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2 +
3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

mupad [B] time = 0.30, size = 80, normalized size = 0.84

$$\frac{3645\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{131072}+\frac{15\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}{512}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(5/2),x)
```

```
[Out] (3645*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/131072 + (15*(4*x + 3i/2)*(x*
3i + 4*x^2)^(3/2))/512 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/24 + (405*(x/2
 + 3i/16)*(x*3i + 4*x^2)^(1/2))/2048
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(4x^2+3ix\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(5/2), x)
```

3.4 $\int \left(3ix+4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {612, 619, 215}

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2
)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{3/2} dx = \frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{27}{64}\int \sqrt{3ix+4x^2}\,dx$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243\int \frac{1}{\sqrt{3ix+4x^2}}dx}{2048}$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{81\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}dx,x,3i+8x\right)}{4096}$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.10

$$\frac{\sqrt{x(4x+3i)}\left(2048x^3+2304ix^2-144x-\frac{243\sqrt[4]{-1}\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}}+162i\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(162*I - 144*x + (2304*I)*x^2 + 2048*x^3 - (243*(-1)^(
1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/2048
```

IntegrateAlgebraic [A] time = 0.18, size = 68, normalized size = 0.99

$$\frac{\sqrt{4x^2+3ix}\left(1024x^3+1152ix^2-72x+81i\right)}{1024}-\frac{243\log\left(4\sqrt{4x^2+3ix}-8x-3i\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (Sqrt[(3*I)*x + 4*x^2]*(81*I - 72*x + (1152*I)*x^2 + 1024*x^3))/1024 - (243
*Log[-3*I - 8*x + 4*Sqrt[(3*I)*x + 4*x^2]])/4096
```

fricas [A] time = 0.41, size = 49, normalized size = 0.71

$$\frac{1}{32768}\left(32768\,x^3+36864i\,x^2-2304\,x+2592i\right)\sqrt{4\,x^2+3i\,x}-\frac{243}{4096}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{567}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32768*(32768*x^3 + 36864*I*x^2 - 2304*x + 2592*I)*sqrt(4*x^2 + 3*I*x) - 2
43/4096*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768
```

giac [A] time = 0.61, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 0
```

maple [A] time = 0.12, size = 51, normalized size = 0.74

$$\frac{243 \operatorname{arcsinh}\left(\frac{8x}{3}+i\right)}{4096}+\frac{(8x+3i)\left(4x^2+3ix\right)^{\frac{3}{2}}}{32}+\frac{27\,(8x+3i)\,\sqrt{4x^2+3ix}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*I*x)^(3/2),x)
```

```
[Out] 1/32*(8*x+3*I)*(4*x^2+3*I*x)^(3/2)+27/1024*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+24
3/4096*arcsinh(8/3*x+I)
```

maxima [A] time = 2.98, size = 76, normalized size = 1.10

$$\frac{1}{4}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{4\,x^2+3i\,x}\,x+\frac{81}{1024}i\sqrt{4\,x^2+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4*
x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

mupad [B] time = 0.16, size = 60, normalized size = 0.87

$$\frac{243\,\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{4096}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}{16}+\frac{27\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(3/2),x)
```

```
[Out] (243*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/4096 + ((4*x + 3i/2)*(x*3i + 4
*x^2)^(3/2))/16 + (27*(x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/64
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4x^2 + 3ix\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(3/2), x)
```

3.5 $\int \sqrt{3ix+4x^2}\,dx$

Optimal. Leaf size=43

$$\frac{1}{16}\sqrt{4x^2+3ix}\,(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {612, 619, 215}

$$\frac{1}{16}\sqrt{4x^2+3ix}\,(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{9}{32}\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$$
$$= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{3}{64}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx, x, 3i+8x\right)$$
$$= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Mathematica [A] time = 0.05, size = 64, normalized size = 1.49

$$\frac{1}{32}\sqrt{x(4x+3i)}\left(16x - \frac{9\sqrt[4]{-1}\,\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}} + 6i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*S
qrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x])))/32
```

IntegrateAlgebraic [A] time = 0.10, size = 56, normalized size = 1.30

$$\frac{1}{16}(8x+3i)\sqrt{4x^2+3ix} - \frac{9}{64}\log\left(4\sqrt{4x^2+3ix} - 8x - 3i\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 - (9*Log[-3*I - 8*x + 4*Sqrt[(3*I)*x
 + 4*x^2]])/64
```

fricas [A] time = 0.41, size = 39, normalized size = 0.91

$$\frac{1}{256}\sqrt{4\,x^2+3i\,x}\,(128\,x+48i) - \frac{9}{64}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x} - \frac{3}{4}i\right) - \frac{9}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/256*sqrt(4*x^2 + 3*I*x)*(128*x + 48*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I
*x) - 3/4*I) - 9/256
```

giac [A] time = 0.63, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 0
```

maple [A] time = 0.10, size = 31, normalized size = 0.72

$$\frac{9 \operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{64} + \frac{(8x + 3i)\sqrt{4x^2 + 3ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*I*x)^(1/2),x)
```

```
[Out] 1/16*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+9/64*arcsinh(8/3*x+I)
```

maxima [A] time = 2.96, size = 49, normalized size = 1.14

$$\frac{1}{2}\sqrt{4x^2 + 3ix}\,x + \frac{3}{16}i\sqrt{4x^2 + 3ix} + \frac{9}{64}\log\left(8x + 4\sqrt{4x^2 + 3ix} + 3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*s
qrt(4*x^2 + 3*I*x) + 3*I)
```

mupad [B] time = 0.09, size = 39, normalized size = 0.91

$$\frac{9\ln\left(x + \frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2} + \frac{3}{8}\mathrm{i}\right)}{64} + \left(\frac{x}{2} + \frac{3}{16}\mathrm{i}\right)\sqrt{4\,x^2 + x\,3\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(1/2),x)
```

```
[Out] (9*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/64 + (x/2 + 3i/16)*(x*3i + 4*x^2
)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4x^2 + 3ix}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 3*I*x), x)
```

3.6 $\int \left(3x-4x^2\right)^{7/2} \, dx$

Optimal. Leaf size=101

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{2296}{}$$

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{229635\sin^{-1}\left(1-\frac{8x}{3}\right)}{16777216}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*x - 4*x^2)
^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/2048 - ((3 - 8*x)*(3*x
- 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)/3])/16777216
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{7/2} dx = -\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}+\frac{63}{128}\int \left(3x-4x^2\right)^{5/2} dx$$
$$= -\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}+\frac{945\int \left(3x-4x^2\right)^{3/2} dx}{4096}$$
$$= -\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}+\frac{25515}{}$$
$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3$$

$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3$$
$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.87

$$\frac{2x\left(134217728x^8-452984832x^7+581959680x^6-338558976x^5+75534336x^4+41472x^3+54432x^2+102060x-229635\right)-229635\sqrt{3-4x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{8388608\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (2*x*(-229635 + 102060*x + 54432*x^2 + 41472*x^3 + 75534336*x^4 - 338558976
*x^5 + 581959680*x^6 - 452984832*x^7 + 134217728*x^8) - 229635*Sqrt[3 - 4*x
]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(8388608*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.31, size = 78, normalized size = 0.77

$$\frac{\sqrt{3x-4x^2}\left(-33554432x^7+88080384x^6-79429632x^5+25067520x^4-82944x^3-72576x^2-68040x-76545\right)}{4194304}-\frac{229635\tan^{-1}\left(\frac{\sqrt{3x-4x^2}}{2x}\right)}{8388608}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[3*x - 4*x^2]*(-76545 - 68040*x - 72576*x^2 - 82944*x^3 + 25067520*x^4
 - 79429632*x^5 + 88080384*x^6 - 33554432*x^7))/4194304 - (229635*ArcTan[Sq
rt[3*x - 4*x^2]/(2*x)])/8388608
```

fricas [A] time = 0.40, size = 68, normalized size = 0.67

$$-\frac{1}{4194304}\left(33554432\,x^7-88080384\,x^6+79429632\,x^5-25067520\,x^4+82944\,x^3+72576\,x^2+68040\,x+76545\right)\sqrt{-4\,x^2+3\,x}-\frac{229635}{8388608}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520*x^4 + 829
44*x^3 + 72576*x^2 + 68040*x + 76545)*sqrt(-4*x^2 + 3*x) - 229635/8388608*a
rctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

giac [A] time = 0.47, size = 57, normalized size = 0.56

$$-\frac{1}{4194304}\left(8\left(16\left(8\left(32\left(8\left(16\left(8\,x-21\right)x+303\right)x-765\right)x+81\right)x+567\right)x+8505\right)x+76545\right)\sqrt{-4\,x^2+3\,x}+\frac{229635}{16777216}\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x + 81)*x + 56
7)*x + 8505)*x + 76545)*sqrt(-4*x^2 + 3*x) + 229635/16777216*arcsin(8/3*x -
 1)
```

maple [A] time = 0.04, size = 82, normalized size = 0.81

$$\frac{229635\arcsin\left(\frac{8x}{3}-1\right)}{16777216}-\frac{945\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{3}{2}}}{131072}-\frac{21\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{5}{2}}}{2048}-\frac{\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{7}{2}}}{64}-\frac{25515\left(-8x+3\right)\sqrt{-4x^2+3x}}{4194304}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(7/2),x)
```

```
[Out] -945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-1
/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-1+8/3*x)-25515/41943
04*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

maxima [A] time = 3.06, size = 117, normalized size = 1.16

$$\frac{1}{8}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}x-\frac{3}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}+\frac{21}{256}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{63}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{2835}{131072}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{-4\,x^2+3\,x}\,x-\frac{76545}{4194304}\sqrt{-4\,x^2+3\,x}-\frac{229635}{16777216}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*x^2 + 3*x)^(7/2)*x - 3/64*(-4*x^2 + 3*x)^(7/2) + 21/256*(-4*x^2 + 3
*x)^(5/2)*x - 63/2048*(-4*x^2 + 3*x)^(5/2) + 945/16384*(-4*x^2 + 3*x)^(3/2)
```

```
*x - 2835/131072*(-4*x^2 + 3*x)^(3/2) + 25515/524288*sqrt(-4*x^2 + 3*x)*x -
 76545/4194304*sqrt(-4*x^2 + 3*x) - 229635/16777216*arcsin(-8/3*x + 1)
```

mupad [B] time = 0.17, size = 81, normalized size = 0.80

$$\frac{229635\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{16777216}+\frac{945\,\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{3/2}}{65536}+\frac{21\,\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{5/2}}{1024}+\frac{\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{7/2}}{32}+\frac{25515\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(7/2),x)
```

```
[Out] (229635*asin((8*x)/3 - 1))/16777216 + (945*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))
/65536 + (21*(4*x - 3/2)*(3*x - 4*x^2)^(5/2))/1024 + ((4*x - 3/2)*(3*x - 4*
x^2)^(7/2))/32 + (25515*(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/262144
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4x^2+3x\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(7/2), x)
```

3.7 $\int \left(3x-4x^2\right)^{5/2} dx$

Optimal. Leaf size=79

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4*x^2)^(3/2
))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*ArcSin[1 - (8*x)/3])/1
31072
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{5/2} dx = -\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{15}{32}\int \left(3x-4x^2\right)^{3/2} dx$$
$$= -\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{405\int \sqrt{3x-4x^2}\, dx}{2048}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} + \frac{3645\int -}{65}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} - \frac{1215\,\mathrm{Sub}}{}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024} - \frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2} - \frac{3645\sin}{13}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.99

$$\frac{2x\left(-1048576x^6+2752512x^5-2469888x^4+760320x^3+2592x^2+4860x-10935\right)-10935\sqrt{3-4x}\,\sqrt{x}\,\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{196608\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (2*x*(-10935 + 4860*x + 2592*x^2 + 760320*x^3 - 2469888*x^4 + 2752512*x^5 -
 1048576*x^6) - 10935*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(196
608*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.22, size = 68, normalized size = 0.86

$$\frac{\sqrt{3x-4x^2}\left(262144x^5-491520x^4+248832x^3-3456x^2-3240x-3645\right)}{98304} - \frac{3645\tan^{-1}\left(\frac{\sqrt{3x-4x^2}}{2x}\right)}{65536}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[3*x - 4*x^2]*(-3645 - 3240*x - 3456*x^2 + 248832*x^3 - 491520*x^4 + 2
62144*x^5))/98304 - (3645*ArcTan[Sqrt[3*x - 4*x^2]/(2*x)])/65536
```

fricas [A] time = 0.39, size = 58, normalized size = 0.73

$$\frac{1}{98304}\left(262144\,x^5-491520\,x^4+248832\,x^3-3456\,x^2-3240\,x-3645\right)\sqrt{-4\,x^2+3\,x}-\frac{3645}{65536}\,\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*s
qrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

giac [A] time = 0.45, size = 47, normalized size = 0.59

$$\frac{1}{98304}\,(8\,(16\,(8\,(32\,(8\,x-15)x+243)x-27)x-405)x-3645)\sqrt{-4\,x^2+3\,x}+\frac{3645}{131072}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(-
4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

maple [A] time = 0.05, size = 64, normalized size = 0.81

$$\frac{3645\arcsin\left(\frac{8x}{3}-1\right)}{131072}-\frac{15\,(-8x+3)\left(-4x^2+3x\right)^{\frac{3}{2}}}{1024}-\frac{(-8x+3)\left(-4x^2+3x\right)^{\frac{5}{2}}}{48}-\frac{405\,(-8x+3)\,\sqrt{-4x^2+3x}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(5/2),x)
```

```
[Out] -15/1024*(-8*x+3)*(-4*x^2+3*x)^(3/2)-1/48*(-8*x+3)*(-4*x^2+3*x)^(5/2)+3645/
131072*arcsin(8/3*x-1)-405/32768*(-8*x+3)*(-4*x^2+3*x)^(1/2)
```

maxima [A] time = 2.94, size = 90, normalized size = 1.14

$$\frac{1}{6}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{1}{16}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{15}{128}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{45}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\,\sqrt{-4\,x^2+3\,x}\,x-\frac{1215}{32768}\,\sqrt{-4\,x^2+3\,x}-\frac{3645}{131072}\,\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 + 3
*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*x
 - 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)
```

mupad [B] time = 0.24, size = 63, normalized size = 0.80

$$\frac{3645\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{131072}+\frac{15\,\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{3/2}}{512}+\frac{\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(5/2),x)
```

```
[Out] (3645*asin((8*x)/3 - 1))/131072 + (15*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))/512
+ ((4*x - 3/2)*(3*x - 4*x^2)^(5/2))/24 + (405*(x/2 - 3/16)*(3*x - 4*x^2)^(1
/2))/2048
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4x^2+3x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(5/2), x)
```

3.8 $\int \left(3x-4x^2\right)^{3/2} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(3/2))/32
 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{3/2} dx = -\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} + \frac{27}{64}\int \sqrt{3x-4x^2}\, dx$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} + \frac{243\int \frac{1}{\sqrt{3x-4x^2}}\, dx}{2048}$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} - \frac{81\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\, dx, x, 3-8x\right)}{4096}$$
$$= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2} - \frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.19

$$\frac{2x\left(4096x^4-7680x^3+3744x^2+108x-243\right)-243\sqrt{3-4x}\,\sqrt{x}\,\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{2048\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (2*x*(-243 + 108*x + 3744*x^2 - 7680*x^3 + 4096*x^4) - 243*Sqrt[3 - 4*x]*Sq
rt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(2048*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.15, size = 58, normalized size = 1.02

$$\frac{\sqrt{3x-4x^2}\left(-1024x^3+1152x^2-72x-81\right)}{1024} - \frac{243\tan^{-1}\left(\frac{\sqrt{3x-4x^2}}{2x}\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3*x - 4*x^2]*(-81 - 72*x + 1152*x^2 - 1024*x^3))/1024 - (243*ArcTan[S
qrt[3*x - 4*x^2]/(2*x)])/2048
```

fricas [A] time = 0.41, size = 48, normalized size = 0.84

$$-\frac{1}{1024}\left(1024\,x^3-1152\,x^2+72\,x+81\right)\sqrt{-4\,x^2+3\,x} - \frac{243}{2048}\,\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*arc
tan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

giac [A] time = 0.52, size = 37, normalized size = 0.65

$$-\frac{1}{1024}\left(8\left(16\left(8x-9\right)x+9\right)x+81\right)\sqrt{-4x^2+3x}+\frac{243}{4096}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcsi
n(8/3*x - 1)
```

maple [A] time = 0.05, size = 46, normalized size = 0.81

$$\frac{243\arcsin\left(\frac{8x}{3}-1\right)}{4096}-\frac{\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{3}{2}}}{32}-\frac{27\left(-8x+3\right)\sqrt{-4x^2+3x}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(3/2),x)
```

```
[Out] -1/32*(-8*x+3)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(8/3*x-1)-27/1024*(-8*x+3)
*(-4*x^2+3*x)^(1/2)
```

maxima [A] time = 2.90, size = 63, normalized size = 1.11

$$\frac{1}{4}\left(-4x^2+3x\right)^{\frac{3}{2}}x-\frac{3}{32}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{-4x^2+3x}\,x-\frac{81}{1024}\sqrt{-4x^2+3x}-\frac{243}{4096}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^2
 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)
```

mupad [B] time = 0.11, size = 45, normalized size = 0.79

$$\frac{243\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{4096}+\frac{\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{3/2}}{16}+\frac{27\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(3/2),x)
```

```
[Out] (243*asin((8*x)/3 - 1))/4096 + ((4*x - 3/2)*(3*x - 4*x^2)^(3/2))/16 + (27*(
x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/64
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4x^2 + 3x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(3/2), x)
```

3.9 $\int \sqrt{3x-4x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{16}\sqrt{3x-4x^2}\,(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{16}\sqrt{3x-4x^2}\,(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3x-4x^2}\,dx = -\frac{1}{16}(3-8x)\sqrt{3x-4x^2} + \frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}\,dx$$
$$= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2} - \frac{3}{64}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x\right)$$
$$= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2} - \frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.66

$$\frac{-2x\left(32x^2-36x+9\right)-9\sqrt{3-4x}\,\sqrt{x}\,\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{32\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x - 4*x^2],x]
```

```
[Out] (-2*x*(9 - 36*x + 32*x^2) - 9*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3
]])/(32*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.09, size = 48, normalized size = 1.37

$$\frac{1}{16}(8x-3)\sqrt{3x-4x^2} - \frac{9}{32}\tan^{-1}\left(\frac{\sqrt{3x-4x^2}}{2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3*x - 4*x^2],x]
```

```
[Out] ((-3 + 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcTan[Sqrt[3*x - 4*x^2]/(2*x)])/32
```

fricas [A] time = 0.42, size = 38, normalized size = 1.09

$$\frac{1}{16}\sqrt{-4x^2+3x}\,(8x-3) - \frac{9}{32}\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

giac [A] time = 0.48, size = 27, normalized size = 0.77

$$\frac{1}{16}\sqrt{-4x^2+3x}(8x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="giac")
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

maple [A] time = 0.05, size = 28, normalized size = 0.80

$$\frac{9\arcsin\left(\frac{8x}{3}-1\right)}{64}-\frac{(-8x+3)\sqrt{-4x^2+3x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(1/2),x)
[Out] 9/64*arcsin(8/3*x-1)-1/16*(-8*x+3)*(-4*x^2+3*x)^(1/2)
```

maxima [A] time = 2.88, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-4x^2+3x}\,x-\frac{3}{16}\sqrt{-4x^2+3x}-\frac{9}{64}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x + 1
)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{9\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{64}+\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(1/2),x)
[Out] (9*asin((8*x)/3 - 1))/64 + (x/2 - 3/16)*(3*x - 4*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4x^2 + 3x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 + 3*x), x)
```

3.10 $\int \sqrt{6x-x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{6x-x^2}\,(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{2}\sqrt{6x-x^2}\,(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6*x - x^2],x]
```

```
[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{6x-x^2}\,dx &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}+\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}\,dx\\ &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,6-2x\right)\\ &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.91

$$\frac{1}{2}(x-3)\sqrt{-((x-6)x)}-9\sin^{-1}\left(\sqrt{1-\frac{x}{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6*x - x^2],x]
```

```
[Out] ((-3 + x)*Sqrt[-((-6 + x)*x)])/2 - 9*ArcSin[Sqrt[1 - x/6]]
```

IntegrateAlgebraic [A] time = 0.11, size = 41, normalized size = 1.17

$$\frac{1}{2}(x-3)\sqrt{6x-x^2}-9\tan^{-1}\left(\frac{\sqrt{6x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[6*x - x^2],x]
```

```
[Out] ((-3 + x)*Sqrt[6*x - x^2])/2 - 9*ArcTan[Sqrt[6*x - x^2]/x]
```

fricas [A] time = 0.40, size = 35, normalized size = 1.00

$$\frac{1}{2}\sqrt{-x^2+6\,x}\,(x-3)-9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

giac [A] time = 0.36, size = 25, normalized size = 0.71

$$\frac{1}{2}\sqrt{-x^2+6x}(x-3)+\frac{9}{2}\arcsin\left(\frac{1}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

maple [A] time = 0.05, size = 28, normalized size = 0.80

$$\frac{9\arcsin\left(\frac{x}{3}-1\right)}{2}-\frac{(-2x+6)\sqrt{-x^2+6x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+6*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

maxima [A] time = 2.97, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+6x}\,x-\frac{3}{2}\sqrt{-x^2+6x}-\frac{9}{2}\arcsin\left(-\frac{1}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{9\operatorname{asin}\left(\frac{x}{3}-1\right)}{2}+\left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{6x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2)^(1/2),x)
```

```
[Out] (9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+6x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + 6*x), x)
```

3.11 $\int \sqrt{5x-9x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{36}\sqrt{5x-9x^2}\,(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{36}\sqrt{5x-9x^2}\,(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5*x - 9*x^2],x]
```

```
[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/216
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{5x-9x^2}\,dx = -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} + \frac{25}{72}\int \frac{1}{\sqrt{5x-9x^2}}\,dx$$
$$= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{5}{216}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25}}}\,dx, x, 5-18x\right)$$
$$= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.66

$$\frac{-3x\left(162x^2-135x+25\right)-25\sqrt{5-9x}\,\sqrt{x}\,\sin^{-1}\left(\sqrt{1-\frac{9x}{5}}\right)}{108\sqrt{-x(9x-5)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5*x - 9*x^2],x]
```

```
[Out] (-3*x*(25 - 135*x + 162*x^2) - 25*Sqrt[5 - 9*x]*Sqrt[x]*ArcSin[Sqrt[1 - (9*
x)/5]])/(108*Sqrt[-(x*(-5 + 9*x))])
```

IntegrateAlgebraic [A] time = 0.11, size = 48, normalized size = 1.37

$$\frac{1}{36}(18x-5)\sqrt{5x-9x^2} - \frac{25}{108}\tan^{-1}\left(\frac{\sqrt{5x-9x^2}}{3x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[5*x - 9*x^2],x]
```

```
[Out] ((-5 + 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcTan[Sqrt[5*x - 9*x^2]/(3*x)])/1
08
```

fricas [A] time = 0.41, size = 38, normalized size = 1.09

$$\frac{1}{36}\sqrt{-9\,x^2+5\,x}\,(18\,x-5) - \frac{25}{108}\,\arctan\left(\frac{\sqrt{-9\,x^2+5\,x}}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^2 + 5*x)/x
)
```

giac [A] time = 0.44, size = 27, normalized size = 0.77

$$\frac{1}{36}\sqrt{-9\,x^2+5\,x}\,(18\,x-5)+\frac{25}{216}\arcsin\left(\frac{18}{5}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

maple [A] time = 0.04, size = 28, normalized size = 0.80

$$\frac{25\arcsin\left(\frac{18x}{5}-1\right)}{216}-\frac{(-18x+5)\sqrt{-9x^2+5x}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9*x^2+5*x)^(1/2),x)
```

```
[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

maxima [A] time = 2.93, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-9\,x^2+5\,x}\,x-\frac{5}{36}\sqrt{-9\,x^2+5\,x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsin(-18/5*x
+ 1)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{25\,\mathrm{asin}\left(\frac{18\,x}{5}-1\right)}{216}+\left(\frac{x}{2}-\frac{5}{36}\right)\sqrt{5\,x-9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 9*x^2)^(1/2),x)
```

```
[Out] (25*asin((18*x)/5 - 1))/216 + (x/2 - 5/36)*(5*x - 9*x^2)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-9x^2 + 5x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x**2+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-9*x**2 + 5*x), x)
```

3.12 $\int \left(x-x^2\right)^{3/2} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 619, 216}

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)^(3/2),x]
```

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 - (3*ArcSin
[1 - 2*x])/128
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x-x^2\right)^{3/2}\,dx &= -\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}+\frac{3}{16}\int \sqrt{x-x^2}\,dx\\ &= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}+\frac{3}{128}\int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\ &= -\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.86

$$\frac{1}{64}\left(-\sqrt{-((x-1)x)}\left(16x^3-24x^2+2x+3\right)-3\sin^{-1}\left(\sqrt{1-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[-((-1 + x)*x)]*(3 + 2*x - 24*x^2 + 16*x^3)) - 3*ArcSin[Sqrt[1 - x]]
)/64
```

IntegrateAlgebraic [A] time = 0.15, size = 51, normalized size = 1.00

$$\frac{1}{64}\sqrt{x-x^2}\left(-16x^3+24x^2-2x-3\right)-\frac{3}{64}\tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - x^2)^(3/2),x]
```

```
[Out] (Sqrt[x - x^2]*(-3 - 2*x + 24*x^2 - 16*x^3))/64 - (3*ArcTan[Sqrt[x - x^2]/x
])/64
```

fricas [A] time = 0.40, size = 43, normalized size = 0.84

$$-\frac{1}{64}\left(16\,x^3-24\,x^2+2\,x+3\right)\sqrt{-x^2+x}-\frac{3}{64}\,\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sqrt(-x^2 +
x)/x)
```

giac [A] time = 0.54, size = 35, normalized size = 0.69

$$-\frac{1}{64}\left(2\left(4\left(2x-3\right)x+1\right)x+3\right)\sqrt{-x^2+x}+\frac{3}{128}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

maple [A] time = 0.04, size = 42, normalized size = 0.82

$$\frac{3\arcsin\left(2x-1\right)}{128}-\frac{\left(-2x+1\right)\left(-x^2+x\right)^{\frac{3}{2}}}{8}-\frac{3\left(-2x+1\right)\sqrt{-x^2+x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(3/2),x)
```

```
[Out] -1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(2*x-1)-3/64*(1-2*x)*(-x^2+x)^(1/2)
```

maxima [A] time = 2.89, size = 55, normalized size = 1.08

$$\frac{1}{4}\left(-x^2+x\right)^{\frac{3}{2}}x-\frac{1}{8}\left(-x^2+x\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^2+x}\,x-\frac{3}{64}\sqrt{-x^2+x}+\frac{3}{128}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/6
4*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

mupad [B] time = 0.19, size = 39, normalized size = 0.76

$$\frac{3\,\mathrm{asin}\left(2x-1\right)}{128}+\frac{3\sqrt{x-x^2}\left(\frac{x}{2}-\frac{1}{4}\right)}{16}+\frac{\left(x-x^2\right)^{3/2}\left(x-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^(3/2),x)
```

```
[Out] (3*asin(2*x - 1))/128 + (3*(x - x^2)^(1/2)*(x/2 - 1/4))/16 + ((x - x^2)^(3/
2)*(x - 1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^2 + x\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(3/2),x)
```

```
[Out] Integral((-x**2 + x)**(3/2), x)
```

3.13 $\int \sqrt{4x+x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*x + x^2],x]
```

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{4x+x^2}\,dx &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-2\int \frac{1}{\sqrt{4x+x^2}}\,dx\\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{4x+x^2}}\right)\\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2}-4\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.14

$$\frac{1}{2}\sqrt{x(x+4)}\left(x-\frac{8\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x+4}\sqrt{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4*x + x^2],x]
[Out] (Sqrt[x*(4 + x)]*(2 + x - (8*ArcSinh[Sqrt[x]/2])/(Sqrt[x]*Sqrt[4 + x])))/2
```

IntegrateAlgebraic [A] time = 0.10, size = 37, normalized size = 1.06

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{\sqrt{x^2+4x}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[4*x + x^2],x]
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[Sqrt[4*x + x^2]/x]
```

fricas [A] time = 0.41, size = 32, normalized size = 0.91

$$\frac{1}{2}\sqrt{x^2+4x}\,(x+2)+2\log\left(-x+\sqrt{x^2+4x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)
```

giac [A] time = 0.52, size = 33, normalized size = 0.94

$$\frac{1}{2}\sqrt{x^2+4\,x}\,(x+2)+2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

maple [A] time = 0.05, size = 33, normalized size = 0.94

$$-2\ln\left(x+2+\sqrt{x^2+4x}\right)+\frac{(2x+4)\sqrt{x^2+4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x)^(1/2),x)
```

```
[Out] 1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(x+2+(x^2+4*x)^(1/2))
```

maxima [A] time = 1.42, size = 41, normalized size = 1.17

$$\frac{1}{2}\sqrt{x^2+4\,x}\,x+\sqrt{x^2+4\,x}-2\,\log\left(2\,x+2\,\sqrt{x^2+4\,x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) + 4
)
```

mupad [B] time = 0.20, size = 29, normalized size = 0.83

$$\sqrt{x^2+4\,x}\,\left(\frac{x}{2}+1\right)-2\,\ln\left(x+\sqrt{x\,(x+4)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2)^(1/2),x)
```

```
[Out] (4*x + x^2)^(1/2)*(x/2 + 1) - 2*log(x + (x*(x + 4))^(1/2) + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2+4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4*x), x)
```

3.14 $\int \sqrt{-8x+x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{x^2-8x}\,(4-x)-16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$-\frac{1}{2}\sqrt{x^2-8x}\,(4-x)-16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8*x + x^2],x]
```

```
[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \sqrt{-8x+x^2}\,dx = -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 8\int \frac{1}{\sqrt{-8x+x^2}}\,dx$$
$$= -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 16\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-8x+x^2}}\right)$$
$$= -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 16\tanh^{-1}\left(\frac{x}{\sqrt{-8x+x^2}}\right)$$

Mathematica [A] time = 0.04, size = 48, normalized size = 1.30

$$\frac{x\left(x^2-12x+32\right)+32\sqrt{-((x-8)x)}\,\sin^{-1}\left(\sqrt{1-\frac{x}{8}}\right)}{2\sqrt{(x-8)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-8*x + x^2],x]
[Out] (x*(32 - 12*x + x^2) + 32*Sqrt[-((-8 + x)*x)]*ArcSin[Sqrt[1 - x/8]])/(2*Sqr
t[(-8 + x)*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 39, normalized size = 1.05

$$\frac{1}{2}(x-4)\sqrt{x^2-8x} - 16\tanh^{-1}\left(\frac{\sqrt{x^2-8x}}{x-8}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-8*x + x^2],x]
[Out] ((-4 + x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[Sqrt[-8*x + x^2]/(-8 + x)]
```

fricas [A] time = 0.38, size = 32, normalized size = 0.86

$$\frac{1}{2}\sqrt{x^2-8\,x}\,(x-4) + 8\,\log\left(-x+\sqrt{x^2-8\,x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)
```

giac [A] time = 0.52, size = 33, normalized size = 0.89

$$\frac{1}{2}\sqrt{x^2-8\,x}\,(x-4)+8\,\log\left(\left|-x+\sqrt{x^2-8\,x}+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))
```

maple [A] time = 0.04, size = 33, normalized size = 0.89

$$-8\ln\left(x-4+\sqrt{x^2-8x}\right)+\frac{(2x-8)\sqrt{x^2-8x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-8*x)^(1/2),x)
```

```
[Out] 1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(-4+x+(x^2-8*x)^(1/2))
```

maxima [A] time = 1.31, size = 43, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2-8\,x}\,x-2\sqrt{x^2-8\,x}-8\,\log\left(2\,x+2\sqrt{x^2-8\,x}-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x) -
 8)
```

mupad [B] time = 0.11, size = 29, normalized size = 0.78

$$\left(\frac{x}{2}-2\right)\sqrt{x^2-8\,x}-8\,\ln\left(x+\sqrt{x\,(x-8)}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 8*x)^(1/2),x)
```

```
[Out] (x/2 - 2)*(x^2 - 8*x)^(1/2) - 8*log(x + (x*(x - 8))^(1/2) - 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - 8*x), x)
```

3.15 $\int \sqrt{-x+x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{4}\sqrt{x^2-x}\,(1-2x)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {612, 620, 206}

$$-\frac{1}{4}\sqrt{x^2-x}\,(1-2x)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^2],x]
```

```
[Out] -((1 - 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \sqrt{-x+x^2}\,dx = -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{8}\int \frac{1}{\sqrt{-x+x^2}}\,dx$$
$$= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$
$$= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)$$

Mathematica [A]   time = 0.03, size = 46, normalized size = 1.18

$$\frac{2x^3-3x^2+x+\sqrt{-((x-1)x)}\,\sin^{-1}\left(\sqrt{1-x}\right)}{4\sqrt{(x-1)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-x + x^2],x]
[Out] (x - 3*x^2 + 2*x^3 + Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(4*Sqrt[(-1 +
 x)*x])
```

IntegrateAlgebraic [A]   time = 0.09, size = 43, normalized size = 1.10

$$\frac{1}{4}(2x-1)\sqrt{x^2-x} - \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x^2-x}}{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^2],x]
[Out] ((-1 + 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[Sqrt[-x + x^2]/(-1 + x)]/4
```

fricas [A]   time = 0.40, size = 36, normalized size = 0.92

$$\frac{1}{4}\sqrt{x^2-x}\,(2\,x-1) + \frac{1}{8}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)
```

giac [A] time = 0.42, size = 37, normalized size = 0.95

$$\frac{1}{4}\sqrt{x^2 - x}(2x - 1) + \frac{1}{8}\log\left(\left|-2x + 2\sqrt{x^2 - x} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
```

maple [A] time = 0.05, size = 33, normalized size = 0.85

$$-\frac{\ln\left(x - \frac{1}{2} + \sqrt{x^2 - x}\right)}{8} + \frac{(2x - 1)\sqrt{x^2 - x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)^(1/2),x)
[Out] 1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(-1/2+x+(x^2-x)^(1/2))
```

maxima [A] time = 1.35, size = 43, normalized size = 1.10

$$\frac{1}{2}\sqrt{x^2 - x}\,x - \frac{1}{4}\sqrt{x^2 - x} - \frac{1}{8}\log\left(2x + 2\sqrt{x^2 - x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) - 1
)
```

mupad [B] time = 0.20, size = 29, normalized size = 0.74

$$\sqrt{x^2 - x}\left(\frac{x}{2} - \frac{1}{4}\right) - \frac{\ln\left(x + \sqrt{x\,(x - 1)} - \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x)^(1/2),x)
[Out] (x^2 - x)^(1/2)*(x/2 - 1/4) - log(x + (x*(x - 1))^(1/2) - 1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)**(1/2),x)
[Out] Integral(sqrt(x**2 - x), x)
```

3.16 $\int \frac{1}{\left(bx+cx^2\right)^{7/2}} \, dx$

Optimal. Leaf size=83

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))/(15*b^4*(
b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqrt[b*x + c*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{1}{\left(bx+cx^2\right)^{7/2}}\,dx = -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}} - \frac{(16c)\int \frac{1}{\left(bx+cx^2\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}} + \frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}} + \frac{\left(128c^2\right)\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx}{15b^4}$$

$$= -\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}} + \frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}} - \frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.84

$$-\frac{2\left(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5\right)}{15b^6(x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
+ 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

IntegrateAlgebraic [A] time = 0.38, size = 82, normalized size = 0.99

$$-\frac{2\sqrt{bx+cx^2}\left(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5\right)}{15b^6x^3(b+cx)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*Sqrt[b*x + c*x^2]*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^
3 + 640*b*c^4*x^4 + 256*c^5*x^5))/(15*b^6*x^3*(b + c*x)^3)
```

fricas [A] time = 0.39, size = 105, normalized size = 1.27

$$-\frac{2\left(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5\right)\sqrt{cx^2+bx}}{15\left(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10*
b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x
^4 + b^9*x^3)
```

giac [**A**] time = 0.49, size = 74, normalized size = 0.89

$$-\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\left(cx^2+bx\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)*
x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

maple [**A**] time = 0.05, size = 75, normalized size = 0.90

$$-\frac{2\left(cx+b\right)\left(256c^5x^5+640c^4x^4b+480c^3x^3b^2+80c^2x^2b^3-10cx\,b^4+3b^5\right)x}{15\left(c\,x^2+bx\right)^{\frac{7}{2}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(7/2),x)
```

```
[Out] -2/15*(c*x+b)*x*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-1
0*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

maxima [**A**] time = 1.34, size = 111, normalized size = 1.34

$$-\frac{4\,cx}{5\left(cx^2+bx\right)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^4}-\frac{512\,c^3x}{15\sqrt{cx^2+bx}\,b^6}-\frac{2}{5\left(cx^2+bx\right)^{\frac{5}{2}}b}+\frac{32\,c}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\sqrt{cx^2+bx}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
- 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32/1
5*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

mupad [B] time = 0.27, size = 96, normalized size = 1.16

$$-\frac{6\,b^5+256\,b\,c^2\,{\left(c\,x^2+b\,x\right)}^2+512\,c^3\,x\,{\left(c\,x^2+b\,x\right)}^2-32\,b^3\,c\,\left(c\,x^2+b\,x\right)+12\,b^4\,c\,x-64\,b^2\,c^2\,x\,\left(c\,x^2+b\,x\right)}{15\,b^6\,{\left(c\,x^2+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(7/2),x)
```

```
[Out] -(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c*
(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c*x
^2)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+cx^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/2), x)
```

3.17 $\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {619, 215}

$$\frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3ix+4x^2}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx, x, 3i+8x\right)\\ &= \frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 53, normalized size = 3.31

$$-\frac{(-1)^{3/4}\sqrt{3-4ix}\,\sqrt{x}\,\sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] -(((-1)^(3/4)*Sqrt[3 - (4*I)*x]*Sqrt[x]*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/
Sqrt[x*(3*I + 4*x)])
```

IntegrateAlgebraic [A] time = 0.10, size = 29, normalized size = 1.81

$$-\frac{1}{2}\log\left(4\sqrt{4x^2+3ix}-8x-3i\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] -1/2*Log[-3*I - 8*x + 4*Sqrt[(3*I)*x + 4*x^2]]
```

fricas [B] time = 0.41, size = 19, normalized size = 1.19

$$-\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [A] time = 0.10, size = 10, normalized size = 0.62

$$\frac{\operatorname{arcsinh}\left(\frac{8x}{3}+i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+3*I*x)^(1/2),x)
```

```
[Out] 1/2*arcsinh(8/3*x+I)
```

maxima [B] time = 2.93, size = 21, normalized size = 1.31

$$\frac{1}{2}\,\log\left(8\,x+4\,\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

mupad [B] time = 0.28, size = 19, normalized size = 1.19

$$\frac{\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(1/2),x)
```

```
[Out] log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{4x^2+3ix}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**2 + 3*I*x), x)
```

3.18 $\int \frac{1}{(3ix+4x^2)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {613}

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-3/2),x]
```

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(3ix+4x^2)^{3/2}} \, dx = \frac{2(3i+8x)}{9\sqrt{3ix+4x^2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$\frac{2(8x+3i)}{9\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-3/2),x]
```

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

IntegrateAlgebraic [A] time = 0.17, size = 38, normalized size = 1.46

$$\frac{2(8x+3i)\sqrt{4x^2+3ix}}{9x(4x+3i)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(-3/2),x]
```

```
[Out] (2*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/(9*x*(3*I + 4*x))
```

fricas [B] time = 0.42, size = 39, normalized size = 1.50

$$\frac{32\,x^2+\sqrt{4\,x^2+3i\,x}\,(16\,x+6i)+24i\,x}{9\left(4\,x^2+3i\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(32*x^2 + sqrt(4*x^2 + 3*I*x)*(16*x + 6*I) + 24*I*x)/(4*x^2 + 3*I*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [A] time = 0.10, size = 21, normalized size = 0.81

$$\frac{\frac{16x}{9}+\frac{2i}{3}}{\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+3*I*x)^(3/2),x)
```

```
[Out] 2/9*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)
```

maxima [A] time = 1.37, size = 28, normalized size = 1.08

$$\frac{16\,x}{9\,\sqrt{4\,x^2+3i\,x}}+\frac{2i}{3\,\sqrt{4\,x^2+3i\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

mupad [B] time = 0.05, size = 20, normalized size = 0.77

$$\frac{16\,x+6\mathrm{i}}{9\,\sqrt{4\,x^2+x\,3\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(3/2),x)
```

```
[Out] (16*x + 6i)/(9*(x*3i + 4*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(4x^2+3ix\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

3.19 $\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\, dx$

Optimal. Leaf size=53

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\left(4x^2+3ix\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {614, 613}

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\left(4x^2+3ix\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(
3*I)*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\,dx &= \frac{2(3i+8x)}{27\left(3ix+4x^2\right)^{3/2}}+\frac{32}{27}\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx\\ &= \frac{2(3i+8x)}{27\left(3ix+4x^2\right)^{3/2}}+\frac{64(3i+8x)}{243\sqrt{3ix+4x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.68

$$\frac{2048x^3 + 2304ix^2 - 432x + 54i}{243(x(4x + 3i))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 50, normalized size = 0.94

$$\frac{2\sqrt{4x^2 + 3ix}\,\left(1024x^3 + 1152ix^2 - 216x + 27i\right)}{243x^2(4x + 3i)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (2*Sqrt[(3*I)*x + 4*x^2]*(27*I - 216*x + (1152*I)*x^2 + 1024*x^3))/(243*x^2
*(3*I + 4*x)^2)
```

fricas [A] time = 0.41, size = 62, normalized size = 1.17

$$\frac{4096\,x^4 + 6144i\,x^3 - 2304\,x^2 + \left(2048\,x^3 + 2304i\,x^2 - 432\,x + 54i\right)\sqrt{4\,x^2 + 3i\,x}}{3888\,x^4 + 5832i\,x^3 - 2187\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (4096*x^4 + 6144*I*x^3 - 2304*x^2 + (2048*x^3 + 2304*I*x^2 - 432*x + 54*I)*
sqrt(4*x^2 + 3*I*x))/(3888*x^4 + 5832*I*x^3 - 2187*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
```

```
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [A] time = 0.10, size = 42, normalized size = 0.79

$$\frac{\frac{16x}{27}+\frac{2i}{9}}{\left(4x^2+3ix\right)^{\frac{3}{2}}}+\frac{\frac{512x}{243}+\frac{64i}{81}}{\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+3*I*x)^(5/2),x)
```

```
[Out] 2/27*(8*x+3*I)/(4*x^2+3*I*x)^(3/2)+64/243*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)
```

maxima [A] time = 1.37, size = 55, normalized size = 1.04

$$\frac{512\,x}{243\,\sqrt{4\,x^2+3i\,x}}+\frac{64i}{81\,\sqrt{4\,x^2+3i\,x}}+\frac{16\,x}{27\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}}+\frac{2i}{9\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x^
2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

mupad [B] time = 0.12, size = 31, normalized size = 0.58

$$\frac{(16\,x+6\mathrm{i})\,\left(128\,x^2+x\,96\mathrm{i}+9\right)}{243\,\left(4\,x^2+x\,3\mathrm{i}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(5/2),x)
```

```
[Out] ((16*x + 6i)*(x*96i + 128*x^2 + 9))/(243*(x*3i + 4*x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(4x^2+3ix\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

3.20 $\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=79

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\left(4x^2+3ix\right)^{3/2}}+\frac{2(8x+3i)}{45\left(4x^2+3ix\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {614, 613}

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\left(4x^2+3ix\right)^{3/2}}+\frac{2(8x+3i)}{45\left(4x^2+3ix\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/(1215*((3*
I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(3*I)*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(3ix+4x^2\right)^{7/2}}\,dx &= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{64}{45}\int \frac{1}{\left(3ix+4x^2\right)^{5/2}}\,dx\\
&= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{128(3i+8x)}{1215\left(3ix+4x^2\right)^{3/2}}+\frac{2048\int \frac{1}{\left(3ix+4x^2\right)^{3/2}}\,dx}{1215}\\
&= \frac{2(3i+8x)}{45\left(3ix+4x^2\right)^{5/2}}+\frac{128(3i+8x)}{1215\left(3ix+4x^2\right)^{3/2}}+\frac{4096(3i+8x)}{10935\sqrt{3ix+4x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.61

$$
\frac{524288x^5+983040ix^4-552960x^3-69120ix^2-6480x+1458i}{10935(x(4x+3i))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^5
)/(10935*(x*(3*I + 4*x))^(5/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 62, normalized size = 0.78

$$
\frac{2\sqrt{4x^2+3ix}\left(262144x^5+491520ix^4-276480x^3-34560ix^2-3240x+729i\right)}{10935x^3(4x+3i)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (2*Sqrt[(3*I)*x + 4*x^2]*(729*I - 3240*x - (34560*I)*x^2 - 276480*x^3 + (49
1520*I)*x^4 + 262144*x^5))/(10935*x^3*(3*I + 4*x)^3)
```

fricas [A] time = 0.41, size = 82, normalized size = 1.04

$$
\frac{1048576\,x^6+2359296i\,x^5-1769472\,x^4-442368i\,x^3+\left(524288\,x^5+983040i\,x^4-552960\,x^3-69120i\,x^2-6480\,x+1458i\right)\sqrt{4\,x^2+3i\,x}}{699840\,x^6+1574640i\,x^5-1180980\,x^4-295245i\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] (1048576*x^6 + 2359296*I*x^5 - 1769472*x^4 - 442368*I*x^3 + (524288*x^5 + 9
83040*I*x^4 - 552960*x^3 - 69120*I*x^2 - 6480*x + 1458*I)*sqrt(4*x^2 + 3*I*
x))/(699840*x^6 + 1574640*I*x^5 - 1180980*x^4 - 295245*I*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [A] time = 0.10, size = 62, normalized size = 0.78

$$\frac{\frac{16x}{45}+\frac{2i}{15}}{\left(4x^2+3ix\right)^{\frac{5}{2}}}+\frac{\frac{1024x}{1215}+\frac{128i}{405}}{\left(4x^2+3ix\right)^{\frac{3}{2}}}+\frac{\frac{32768x}{10935}+\frac{4096i}{3645}}{\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+3*I*x)^(7/2),x)
```

```
[Out] 2/45*(8*x+3*I)/(4*x^2+3*I*x)^(5/2)+128/1215*(8*x+3*I)/(4*x^2+3*I*x)^(3/2)+4
096/10935*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)
```

maxima [A] time = 1.37, size = 82, normalized size = 1.04

$$\frac{32768\,x}{10935\,\sqrt{4\,x^2+3i\,x}}+\frac{4096i}{3645\,\sqrt{4\,x^2+3i\,x}}+\frac{1024\,x}{1215\,\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}}+\frac{128i}{405\,\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}}+\frac{16\,x}{45\,\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}}+\frac{2i}{15\,\left(4\,x^2+3i\,x\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024/
1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/(4
*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)
```

mupad [B] time = 0.29, size = 40, normalized size = 0.51

$$-\frac{-524288\,x^5-x^4\,983040\mathrm{i}+552960\,x^3+x^2\,69120\mathrm{i}+6480\,x-1458\mathrm{i}}{10935\,{\left(x\,\left(4\,x+3\mathrm{i}\right)\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(7/2),x)
```

```
[Out] -(6480*x + x^2*69120i + 552960*x^3 - x^4*983040i - 524288*x^5 - 1458i)/(109
35*(x*(4*x + 3i))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4x^2 + 3ix\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

3.21 $\int \frac{1}{\sqrt{3x-4x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {619, 216}

$$-\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -ArcSin[1 - (8*x)/3]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3x-4x^2}}\,dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x\right)\right)\\ &= -\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 40, normalized size = 3.33

$$-\frac{\sqrt{-x(4x-3)}\,\sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{\sqrt{3-4x}\,\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((Sqrt[-(x*(-3 + 4*x))]*ArcSin[Sqrt[1 - (4*x)/3]])/(Sqrt[3 - 4*x]*Sqrt[x])
)
```

IntegrateAlgebraic [A] time = 0.10, size = 23, normalized size = 1.92

$$-\tan^{-1}\left(\frac{\sqrt{3x-4x^2}}{2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -ArcTan[Sqrt[3*x - 4*x^2]/(2*x)]
```

fricas [B] time = 0.39, size = 19, normalized size = 1.58

$$-\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

giac [A] time = 0.50, size = 8, normalized size = 0.67

$$\frac{1}{2}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(8/3*x - 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.75

$$\frac{\arcsin\left(\frac{8x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(1/2),x)
[Out] 1/2*arcsin(8/3*x-1)
```

maxima [A] time = 2.95, size = 8, normalized size = 0.67

$$-\frac{1}{2}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
[Out] -1/2*arcsin(-8/3*x + 1)
```

mupad [B] time = 0.11, size = 8, normalized size = 0.67

$$\frac{\operatorname{asin}\left(\frac{8x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(1/2),x)
[Out] asin((8*x)/3 - 1)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2+3x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(1/2),x)
[Out] Integral(1/sqrt(-4*x**2 + 3*x), x)
```

3.22 $\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {613}

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-3/2),x]
```

```
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx = -\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{2(8x-3)}{9\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-3/2),x]
```

```
[Out] (2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.15, size = 32, normalized size = 1.45

$$-\frac{2(8x-3)\sqrt{3x-4x^2}}{9x(4x-3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(-3/2),x]
[Out] (-2*(-3 + 8*x)*Sqrt[3*x - 4*x^2])/(9*x*(-3 + 4*x))
```

fricas [A] time = 0.39, size = 29, normalized size = 1.32

$$-\frac{2\sqrt{-4x^2+3x}(8x-3)}{9\left(4x^2-3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

giac [A] time = 0.51, size = 29, normalized size = 1.32

$$-\frac{2\sqrt{-4x^2+3x}(8x-3)}{9\left(4x^2-3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

maple [A] time = 0.05, size = 25, normalized size = 1.14

$$-\frac{2(4x-3)(8x-3)x}{9\left(-4x^2+3x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(3/2),x)
[Out] -2/9*x*(-3+4*x)*(-3+8*x)/(-4*x^2+3*x)^(3/2)
```

maxima [A] time = 1.34, size = 28, normalized size = 1.27

$$\frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}}-\frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")
[Out] 16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

mupad [B] time = 0.14, size = 18, normalized size = 0.82

$$\frac{16\,x-6}{9\,\sqrt{3\,x-4\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(3/2),x)
[Out] (16*x - 6)/(9*(3*x - 4*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-4x^2+3x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(3/2),x)
[Out] Integral((-4*x**2 + 3*x)**(-3/2), x)
```

3.23 $\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*
x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}+\frac{32}{27}\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx\\ &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.69

$$-\frac{2048x^3-2304x^2+432x+54}{243(-x(4x-3))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-5/2),x]
[Out] -1/243*(54 + 432*x - 2304*x^2 + 2048*x^3)/(-(x*(-3 + 4*x)))^(3/2)
```

IntegrateAlgebraic [A] time = 0.18, size = 42, normalized size = 0.93

$$-\frac{2\sqrt{3x-4x^2}\left(1024x^3-1152x^2+216x+27\right)}{243x^2(4x-3)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(-5/2),x]
[Out] (-2*Sqrt[3*x - 4*x^2]*(27 + 216*x - 1152*x^2 + 1024*x^3))/(243*x^2*(-3 + 4*
x)^2)
```

fricas [A] time = 0.41, size = 46, normalized size = 1.02

$$-\frac{2\left(1024\,x^3-1152\,x^2+216\,x+27\right)\sqrt{-4\,x^2+3\,x}}{243\left(16\,x^4-24\,x^3+9\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="fricas")
[Out] -2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*x
^3 + 9*x^2)
```

giac [A] time = 0.41, size = 39, normalized size = 0.87

$$-\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\left(4\,x^2-3\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="giac")
[Out] -2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2
```

maple [A] time = 0.04, size = 35, normalized size = 0.78

$$\frac{2\,(4x-3)\left(1024x^{3}-1152x^{2}+216x+27\right)x}{243\left(-4x^{2}+3x\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(5/2),x)
```

```
[Out] 2/243*x*(4*x-3)*(1024*x^3-1152*x^2+216*x+27)/(-4*x^2+3*x)^(5/2)
```

maxima [A] time = 1.36, size = 55, normalized size = 1.22

$$\frac{512\,x}{243\,\sqrt{-4\,x^{2}+3\,x}}-\frac{64}{81\,\sqrt{-4\,x^{2}+3\,x}}+\frac{16\,x}{27\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}}-\frac{2}{9\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x/(-4*x^2 +
 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

mupad [B] time = 0.03, size = 28, normalized size = 0.62

$$\frac{(16\,x-6)\,\left(-128\,x^{2}+96\,x+9\right)}{243\left(3\,x-4\,x^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(5/2),x)
```

```
[Out] ((16*x - 6)*(96*x - 128*x^2 + 9))/(243*(3*x - 4*x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-4x^{2}+3x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-5/2), x)
```

3.24 $\int \frac{1}{\left(3x-4x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {614, 613}

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(3*x - 4*x^
2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}} + \frac{64}{45}\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$$

$$= -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}} - \frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}} + \frac{2048\int \frac{1}{(3x-4x^2)^{3/2}}\,dx}{1215}$$

$$= -\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}} - \frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}} - \frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.76

$$\frac{2\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935(3-4x)^2x^2\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(109
35*(3 - 4*x)^2*x^2*Sqrt[-(x*(-3 + 4*x))])
```

IntegrateAlgebraic [A] time = 0.23, size = 52, normalized size = 0.78

$$-\frac{2\sqrt{3x-4x^2}\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935x^3(4x-3)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (-2*Sqrt[3*x - 4*x^2]*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4
+ 262144*x^5))/(10935*x^3*(-3 + 4*x)^3)
```

fricas [A] time = 0.40, size = 61, normalized size = 0.91

$$-\frac{2\left(262144\,x^5-491520\,x^4+276480\,x^3-34560\,x^2-3240\,x-729\right)\sqrt{-4\,x^2+3\,x}}{10935\left(64\,x^6-144\,x^5+108\,x^4-27\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)*
sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)
```

giac [A] time = 0.50, size = 49, normalized size = 0.73

$$-\frac{2\,(8\,(32\,(8\,(16\,(8\,x-15)x+135)x-135)x-405)x-729)\sqrt{-4\,x^2+3\,x}}{10935\left(4\,x^2-3\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*sqrt(
-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

maple [A] time = 0.05, size = 45, normalized size = 0.67

$$-\frac{2\,(4x-3)\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)x}{10935\left(-4x^2+3x\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(7/2),x)
```

```
[Out] -2/10935*x*(4*x-3)*(262144*x^5-491520*x^4+276480*x^3-34560*x^2-3240*x-729)/
(-4*x^2+3*x)^(7/2)
```

maxima [A] time = 1.37, size = 82, normalized size = 1.22

$$\frac{32768\,x}{10935\,\sqrt{-4\,x^2+3\,x}}-\frac{4096}{3645\,\sqrt{-4\,x^2+3\,x}}+\frac{1024\,x}{1215\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}}-\frac{128}{405\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}}+\frac{16\,x}{45\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}}-\frac{2}{15\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) + 1024/1215
*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2) + 16/45*x/(-4*x^2 +
3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

mupad [B] time = 0.20, size = 73, normalized size = 1.09

$$-\frac{6480\,x-9216\,x\,\left(3\,x-4\,x^2\right)-32768\,x\left(3\,x-4\,x^2\right)^2+12288\left(3\,x-4\,x^2\right)^2-13824\,x^2+1458}{\left(3\,x-4\,x^2\right)^{3/2}\,\left(32805\,x-43740\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(7/2),x)
```

```
[Out] -(6480*x - 9216*x*(3*x - 4*x^2) - 32768*x*(3*x - 4*x^2)^2 + 12288*(3*x - 4*
x^2)^2 - 13824*x^2 + 1458)/((3*x - 4*x^2)^(3/2)*(32805*x - 43740*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-4x^2+3x\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-7/2), x)
```

3.25 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {619, 216}

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] -(ArcSin[1 - 2*b*x]/b)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, b-2b^2x\right)}{b^2}\\ &= -\frac{\sin^{-1}(1-2bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 47, normalized size = 3.92

$$\frac{2\sqrt{x}\,\sqrt{1-bx}\,\sin^{-1}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,\sqrt{-bx(bx-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 - b*x]*ArcSin[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[-(b*x*(-1 +
 b*x))])
```

IntegrateAlgebraic [B] time = 0.18, size = 97, normalized size = 8.08

$$\frac{\sqrt{-b^2}\log\left(8b^2x^2+8\sqrt{-b^2}\,x\sqrt{bx-b^2x^2}-4bx-1\right)}{2b^2}-\frac{\tan^{-1}\left(2\sqrt{-b^2}\,x-2\sqrt{bx-b^2x^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] -(ArcTan[2*Sqrt[-b^2]*x - 2*Sqrt[b*x - b^2*x^2]]/b) + (Sqrt[-b^2]*Log[-1 -
4*b*x + 8*b^2*x^2 + 8*Sqrt[-b^2]*x*Sqrt[b*x - b^2*x^2]])/(2*b^2)
```

fricas [B] time = 0.41, size = 27, normalized size = 2.25

$$-\frac{2\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b
```

giac [A] time = 0.52, size = 15, normalized size = 1.25

$$-\frac{\arcsin\left(-2\,bx+1\right)\operatorname{sgn}(b)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-2*b*x + 1)*sgn(b)/abs(b)
```

maple [B] time = 0.07, size = 35, normalized size = 2.92

$$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^2*x^2+b*x)^(1/2),x)
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x)^(1/2))
```

maxima [A] time = 2.92, size = 21, normalized size = 1.75

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-(2*b^2*x - b)/b)/b
```

mupad [B] time = 0.31, size = 42, normalized size = 3.50

$$\frac{\ln\left(\frac{\frac{b}{2}-b^2\,x}{\sqrt{-b^2}}+\sqrt{b\,x-b^2\,x^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - b^2*x^2)^(1/2),x)
[Out] log((b/2 - b^2*x)/(-b^2)^(1/2) + (b*x - b^2*x^2)^(1/2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b**2*x**2+b*x)**(1/2),x)
[Out] Integral(1/sqrt(-b**2*x**2 + b*x), x)
```

3.26 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-b^2x^2}\,dx, x, \frac{x}{\sqrt{bx+b^2x^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.88

$$\frac{2\sqrt{x}\sqrt{bx+1}\,\sinh^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{bx(bx+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + b*x]*ArcSinh[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[b*x*(1 + b
*x)])
```

IntegrateAlgebraic [B] time = 0.25, size = 117, normalized size = 4.88

$$-\frac{\log\left(2\sqrt{b^2x^2+bx}-2\sqrt{b^2}\,x-1\right)}{2\sqrt{b^2}}-\frac{\log\left(2\sqrt{b^2x^2+bx}-2\sqrt{b^2}\,x+1\right)}{2\sqrt{b^2}}-\frac{\tanh^{-1}\left(2\sqrt{b^2}\,x-2\sqrt{b^2x^2+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] -(ArcTanh[2*Sqrt[b^2]*x - 2*Sqrt[b*x + b^2*x^2]]/b) - Log[-1 - 2*Sqrt[b^2]*
x + 2*Sqrt[b*x + b^2*x^2]]/(2*Sqrt[b^2]) - Log[1 - 2*Sqrt[b^2]*x + 2*Sqrt[b
*x + b^2*x^2]]/(2*Sqrt[b^2])
```

fricas [A] time = 0.38, size = 27, normalized size = 1.12

$$-\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

giac [A] time = 0.55, size = 36, normalized size = 1.50

$$-\frac{\log\left(\left|-2\left(x|b|-\sqrt{b^2x^2+bx}\right)|b|-b\right|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(x*abs(b) - sqrt(b^2*x^2 + b*x))*abs(b) - b))/abs(b)
```

maple [A] time = 0.05, size = 37, normalized size = 1.54

$$\frac{\ln\left(\frac{b^2x+\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+b*x)^(1/2),x)
```

```
[Out] ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.37, size = 29, normalized size = 1.21

$$\frac{\log\left(2\,b^2 x + 2\,\sqrt{b^2 x^2 + bx}\,b + b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x)*b + b)/b
```

mupad [B] time = 0.23, size = 36, normalized size = 1.50

$$\frac{\ln\left(\frac{x\,b^2+\frac{b}{2}}{\sqrt{b^2}}+\sqrt{b^2\,x^2+b\,x}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + b^2*x^2)^(1/2),x)
```

```
[Out] log((b/2 + b^2*x)/(b^2)^(1/2) + (b*x + b^2*x^2)^(1/2))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2 x^2 + bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b**2*x**2 + b*x), x)
```

3.27 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {619, 216}

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[6*x - x^2],x]
```

```
[Out] -ArcSin[1 - x/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{6x-x^2}}\,dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 6-2x\right)\right)\\ &= -\sin^{-1}\left(1-\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-2\sin^{-1}\left(\sqrt{1-\frac{x}{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[6*x - x^2],x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x/6]]
```

IntegrateAlgebraic [A] time = 0.09, size = 20, normalized size = 2.00

$$-2\tan^{-1}\left(\frac{\sqrt{6x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[6*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[6*x - x^2]/x]
```

fricas [B] time = 0.42, size = 18, normalized size = 1.80

$$-2\ \arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + 6*x)/x)
```

giac [A] time = 0.39, size = 6, normalized size = 0.60

$$\arcsin\left(\frac{1}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/3*x - 1)
```

maple [A] time = 0.05, size = 7, normalized size = 0.70

$$\arcsin\left(\frac{x}{3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+6*x)^(1/2),x)
```

```
[Out] arcsin(1/3*x-1)
```

maxima [A] time = 3.03, size = 8, normalized size = 0.80

$$-\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/3*x + 1)
```

mupad [B] time = 0.11, size = 6, normalized size = 0.60

$$\mathrm{asin}\left(\frac{x}{3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x - x^2)^(1/2),x)
[Out] asin(x/3 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+6x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+6*x)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 + 6*x), x)
```

3.28 $\int \frac{1}{\sqrt{4x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {620, 206}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{4x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2\sqrt{x}\,\sqrt{x+4}\,\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x(x+4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[4 + x]*ArcSinh[Sqrt[x]/2])/Sqrt[x*(4 + x)]
```

IntegrateAlgebraic [A] time = 0.08, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{x^2+4x}-x-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[4*x + x^2],x]
```

```
[Out] -Log[-2 - x + Sqrt[4*x + x^2]]
```

fricas [A] time = 0.40, size = 17, normalized size = 1.06

$$-\log\left(-x+\sqrt{x^2+4\,x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x) - 2)
```

giac [A] time = 0.54, size = 18, normalized size = 1.12

$$-\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

maple [A] time = 0.04, size = 14, normalized size = 0.88

$$\ln\left(x+2+\sqrt{x^2+4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x)^(1/2),x)
```

```
[Out] ln(x+2+(x^2+4*x)^(1/2))
```

maxima [A] time = 1.31, size = 17, normalized size = 1.06

$$\log\left(2\,x + 2\,\sqrt{x^2 + 4\,x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

mupad [B] time = 0.51, size = 11, normalized size = 0.69

$$\ln\left(x + \sqrt{x\,\left(x+4\right)} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^2)^(1/2),x)
```

```
[Out] log(x + (x*(x + 4))^(1/2) + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + 4x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x), x)
```

3.29 $\int \frac{1}{\sqrt{-2x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {620, 206}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-2x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{-2x+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2\sqrt{(x-2)x}\,\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)}{\sqrt{-((x-2)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2*x + x^2],x]
[Out] (2*Sqrt[(-2 + x)*x]*ArcSin[Sqrt[1 - x/2]])/Sqrt[-((-2 + x)*x)]
```

IntegrateAlgebraic [**A**] time = 0.08, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{x^2-2x}-x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2*x + x^2],x]
[Out] -Log[1 - x + Sqrt[-2*x + x^2]]
```

fricas [**A**] time = 0.39, size = 17, normalized size = 1.06

$$-\log\left(-x+\sqrt{x^2-2\,x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 2*x) + 1)
```

giac [**A**] time = 0.54, size = 18, normalized size = 1.12

$$-\log\left(\left|-x+\sqrt{x^2-2\,x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 2*x) + 1))
```

maple [**A**] time = 0.05, size = 14, normalized size = 0.88

$$\ln\left(x-1+\sqrt{x^2-2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x)^(1/2),x)
[Out] ln(x-1+(x^2-2*x)^(1/2))
```

maxima [A] time = 1.33, size = 17, normalized size = 1.06

$$\log\left(2\,x + 2\,\sqrt{x^2 - 2\,x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

mupad [B] time = 0.53, size = 11, normalized size = 0.69

$$\ln\left(x + \sqrt{x\,(x-2)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x)^(1/2),x)
[Out] log(x + (x*(x - 2))^(1/2) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 - 2x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x)**(1/2),x)
[Out] Integral(1/sqrt(x**2 - 2*x), x)
```

3.30 $\int \left(a+c x^2\right)^4 \, dx$

Optimal. Leaf size=51

$$a^4 x+\frac{4}{3} a^3 c x^3+\frac{6}{5} a^2 c^2 x^5+\frac{4}{7} a c^3 x^7+\frac{c^4 x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{6}{5} a^2 c^2 x^5+\frac{4}{3} a^3 c x^3+a^4 x+\frac{4}{7} a c^3 x^7+\frac{c^4 x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+c x^2\right)^4 \, dx &= \int \left(a^4+4 a^3 c x^2+6 a^2 c^2 x^4+4 a c^3 x^6+c^4 x^8\right) \, dx \\ &= a^4 x+\frac{4}{3} a^3 c x^3+\frac{6}{5} a^2 c^2 x^5+\frac{4}{7} a c^3 x^7+\frac{c^4 x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 51, normalized size = 1.00

$$a^4 x+\frac{4}{3} a^3 c x^3+\frac{6}{5} a^2 c^2 x^5+\frac{4}{7} a c^3 x^7+\frac{c^4 x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^4,x]
[Out] IntegrateAlgebraic[(a + c*x^2)^4, x]
```

fricas [A] time = 0.35, size = 43, normalized size = 0.84

$$\frac{1}{9}x^9c^4 + \frac{4}{7}x^7c^3a + \frac{6}{5}x^5c^2a^2 + \frac{4}{3}x^3ca^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="fricas")
[Out] 1/9*x^9*c^4 + 4/7*x^7*c^3*a + 6/5*x^5*c^2*a^2 + 4/3*x^3*c*a^3 + x*a^4
```

giac [A] time = 0.39, size = 43, normalized size = 0.84

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="giac")
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

maple [A] time = 0.04, size = 44, normalized size = 0.86

$$\frac{1}{9}c^4x^9 + \frac{4}{7}a\,c^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3c\,x^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^4,x)
[Out] a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

maxima [A] time = 1.31, size = 43, normalized size = 0.84

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

mupad [B] time = 0.03, size = 43, normalized size = 0.84

$$a^4\,x+\frac{4\,a^3\,c\,x^3}{3}+\frac{6\,a^2\,c^2\,x^5}{5}+\frac{4\,a\,c^3\,x^7}{7}+\frac{c^4\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^4,x)
```

```
[Out] a^4*x + (c^4*x^9)/9 + (4*a^3*c*x^3)/3 + (4*a*c^3*x^7)/7 + (6*a^2*c^2*x^5)/5
```

sympy [A] time = 0.07, size = 49, normalized size = 0.96

$$a^{4}x+\frac{4a^{3}cx^{3}}{3}+\frac{6a^{2}c^{2}x^{5}}{5}+\frac{4ac^{3}x^{7}}{7}+\frac{c^{4}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**4,x)
```

```
[Out] a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**9
/9
```

3.31 $\int \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=35

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2cx^3+a^3x+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3,x]
```

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^3 \, dx &= \int \left(a^3+3a^2cx^2+3ac^2x^4+c^3x^6\right) dx\\ &= a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3,x]
```

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + cx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^3,x]
[Out] IntegrateAlgebraic[(a + c*x^2)^3, x]
```

fricas [A] time = 0.35, size = 31, normalized size = 0.89

$$\frac{1}{7}x^7c^3 + \frac{3}{5}x^5c^2a + x^3ca^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="fricas")
[Out] 1/7*x^7*c^3 + 3/5*x^5*c^2*a + x^3*c*a^2 + x*a^3
```

giac [A] time = 0.53, size = 31, normalized size = 0.89

$$\frac{1}{7}\,c^3x^7 + \frac{3}{5}\,ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="giac")
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

maple [A] time = 0.04, size = 32, normalized size = 0.91

$$\frac{1}{7}c^3x^7 + \frac{3}{5}a\,c^2x^5 + a^2c\,x^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3,x)
[Out] a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

maxima [A] time = 1.35, size = 31, normalized size = 0.89

$$\frac{1}{7}\,c^3x^7 + \frac{3}{5}\,ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x+a^2\,c\,x^3+\frac{3\,a\,c^2\,x^5}{5}+\frac{c^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3,x)
```

```
[Out] a^3*x + (c^3*x^7)/7 + a^2*c*x^3 + (3*a*c^2*x^5)/5
```

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^{3}x+a^{2}cx^{3}+\frac{3ac^{2}x^{5}}{5}+\frac{c^{3}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7
```

3.32 $\int \left(a+cx^2\right)^2 dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^2 dx &= \int \left(a^2+2acx^2+c^2x^4\right) dx\\ &= a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^2,x]
[Out] IntegrateAlgebraic[(a + c*x^2)^2, x]
```

fricas [A] time = 0.35, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5c^2 + \frac{2}{3}x^3ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="fricas")
[Out] 1/5*x^5*c^2 + 2/3*x^3*c*a + x*a^2
```

giac [A] time = 0.43, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="giac")
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

maple [A] time = 0.04, size = 22, normalized size = 0.88

$$\frac{1}{5}c^2x^5 + \frac{2}{3}ac\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2,x)
[Out] a^2*x+2/3*a*c*x^3+1/5*c^2*x^5
```

maxima [A] time = 1.36, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,c\,x^3}{3}+\frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2,x)
```

```
[Out] a^2*x + (c^2*x^5)/5 + (2*a*c*x^3)/3
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$a^{2}x+\frac{2acx^{3}}{3}+\frac{c^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**3/3 + c**2*x**5/5
```

3.33 $\int \left(a+c x^2\right) \, dx$

Optimal. Leaf size=12

$$a x+\frac{c x^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$a x+\frac{c x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + c*x^2,x]
[Out] a*x + (c*x^3)/3
```

Rubi steps

$$\int \left(a+c x^2\right) \, dx=a x+\frac{c x^3}{3}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$a x+\frac{c x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + c*x^2,x]
[Out] a*x + (c*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+c x^2\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + c*x^2,x]
[Out] IntegrateAlgebraic[a + c*x^2, x]
```

fricas [A] time = 0.36, size = 10, normalized size = 0.83

$$\frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="fricas")
[Out] 1/3*x^3*c + x*a
```

giac [A] time = 0.35, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="giac")
[Out] 1/3*c*x^3 + a*x
```

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{1}{3}c\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2+a,x)
[Out] a*x+1/3*c*x^3
```

maxima [A] time = 1.29, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="maxima")
[Out] 1/3*c*x^3 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{c\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + c*x^2,x)
```

```
[Out] a*x + (c*x^3)/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+a,x)
```

```
[Out] a*x + c*x**3/3
```

3.34 $\int \frac{1}{a+cx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+cx^2}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)^(-1), x]
```

fricas [A] time = 0.41, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{2\,ac},\ \frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))/(a*c), sqrt(
a*c)*arctan(sqrt(a*c)*x/a)/(a*c)]
```

giac [A] time = 0.40, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

maple [A] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a),x)
```

```
[Out] 1/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

maxima [A] time = 2.90, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

mupad [B] time = 0.07, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2),x)
```

```
[Out] atan((c^(1/2)*x)/a^(1/2))/(a^(1/2)*c^(1/2))
```

sympy [B] time = 0.14, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x)/2 + sqrt(-1/(a*c))*log(a*sqrt(-1
/(a*c)) + x)/2
```

3.35 $\int \frac{1}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^2}\,dx &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\int \frac{1}{a+cx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)^(-2), x]
```

fricas [A] time = 0.42, size = 120, normalized size = 2.67

$$\left[\frac{2\,acx-\left(cx^2+a\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{4\left(a^2c^2x^2+a^3c\right)},\,\frac{acx+\left(cx^2+a\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{2\left(a^2c^2x^2+a^3c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x - (c*x^2 + a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*
x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*x + (c*x^2 + a)*sqrt(a*c)*arctan
(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

giac [A] time = 0.42, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,a}+\frac{x}{2\left(cx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*x/((c*x^2 + a)*a)
```

maple [A] time = 0.05, size = 36, normalized size = 0.80

$$\frac{x}{2\left(c\,x^{2}+a\right)a}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^2,x)
[Out] 1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)
```

maxima [A] time = 3.11, size = 35, normalized size = 0.78

$$\frac{x}{2\left(acx^{2}+a^{2}\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*x/(a*c*x^2 + a^2) + 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a)
```

mupad [B] time = 0.14, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(c\,x^{2}+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^2,x)
[Out] x/(2*a*(a + c*x^2)) + atan((c^(1/2)*x)/a^(1/2))/(2*a^(3/2)*c^(1/2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$\frac{x}{2a^{2}+2acx^{2}}-\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}c}}\log\left(a^{2}\sqrt{-\frac{1}{a^{3}c}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**2,x)
[Out] x/(2*a**2 + 2*a*c*x**2) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x
)/4 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x)/4
```

3.36 $\int \frac{1}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{x}{4a\left(a+cx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x}{4a\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^3}\,dx = \frac{x}{4a\left(a+cx^2\right)^2} + \frac{3\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{4a}$$
$$= \frac{x}{4a\left(a+cx^2\right)^2} + \frac{3x}{8a^2\left(a+cx^2\right)} + \frac{3\int \frac{1}{a+cx^2}\,dx}{8a^2}$$
$$= \frac{x}{4a\left(a+cx^2\right)^2} + \frac{3x}{8a^2\left(a+cx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} + \frac{5ax+3cx^3}{8a^2\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)^(-3), x]
```

fricas [A] time = 0.39, size = 188, normalized size = 3.03

$$\left[\frac{6\,ac^2x^3 + 10\,a^2cx - 3\left(c^2x^4 + 2\,acx^2 + a^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2 - 2\,\sqrt{-ac}\,x - a}{cx^2 + a}\right)}{16\left(a^3c^3x^4 + 2\,a^4c^2x^2 + a^5c\right)}, \frac{3\,ac^2x^3 + 5\,a^2cx + 3\left(c^2x^4 + 2\,acx^2 + a^2\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{8\left(a^3c^3x^4 + 2\,a^4c^2x^2 + a^5c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c^2*x^3 + 10*a^2*c*x - 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(-a*c)*
log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2
 + a^5*c), 1/8*(3*a*c^2*x^3 + 5*a^2*c*x + 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqr
t(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

giac [A] time = 0.34, size = 45, normalized size = 0.73

$$\frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8 \sqrt{ac}\, a^2} + \frac{3\, cx^3 + 5\, ax}{8 \left(cx^2 + a\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c*x^3 + 5*a*x)/((c*x^2 +
 a)^2*a^2)
```

maple [A] time = 0.05, size = 51, normalized size = 0.82

$$\frac{x}{4 \left(c\, x^2 + a\right)^2 a} + \frac{3x}{8 \left(c\, x^2 + a\right) a^2} + \frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8/a^2/(a*c)^(1/2)*arctan(1/(a*c)^
(1/2)*c*x)
```

maxima [A] time = 2.99, size = 58, normalized size = 0.94

$$\frac{3\, cx^3 + 5\, ax}{8 \left(a^2 c^2 x^4 + 2\, a^3 c x^2 + a^4\right)} + \frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8 \sqrt{ac}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*x^3 + 5*a*x)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4) + 3/8*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^2)
```

mupad [B] time = 0.16, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,a}+\frac{3\,c\,x^3}{8\,a^2}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) + (3*c*x^3)/(8*a^2))/(a^2 + c^2*x^4 + 2*a*c*x^2) + (3*atan((c^
(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2))
```

sympy [A] time = 0.34, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{5ax+3cx^3}{8a^4+16a^3cx^2+8a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a**5*
c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x**3)/(8*a**4 + 16*a*
*3*c*x**2 + 8*a**2*c**2*x**4)
```

3.37 $\int \left(a+cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*(a + c*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+cx^2\right)^{5/2} dx &= \frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+cx^2\right)^{3/2} dx \\
&= \frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+cx^2}\, dx \\
&= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+cx^2}}\, dx \\
&= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\, dx, x,\right. \\
&= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.90

$$
\frac{1}{48}\sqrt{a+cx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+33a^2x+26acx^3+8c^2x^5\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a])))/48
```

IntegrateAlgebraic [A] time = 0.09, size = 71, normalized size = 0.85

$$
\frac{1}{48}\sqrt{a+cx^2}\left(33a^2x+26acx^3+8c^2x^5\right)-\frac{5a^3\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{16\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

fricas [A] time = 0.43, size = 146, normalized size = 1.74

$$
\left[\frac{15\,a^3\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,\left(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx\right)\sqrt{cx^2+a}}{96\,c}, -\frac{15\,a^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\left(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx\right)\sqrt{cx^2+a}}{48\,c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c, -1/48*(15*a^3*sq
rt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (8*c^3*x^5 + 26*a*c^2*x^3 + 33*
a^2*c*x)*sqrt(c*x^2 + a))/c]
```

giac [A] time = 0.36, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}}+\frac{1}{48}\left(2\left(4\,c^2x^2+13\,ac\right)x^2+33\,a^2\right)\sqrt{cx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2*(4*c^2*x
^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x
```

maple [A] time = 0.04, size = 66, normalized size = 0.79

$$\frac{5a^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{16\sqrt{c}}+\frac{5\sqrt{c\,x^2+a}\;a^2x}{16}+\frac{5\left(c\,x^2+a\right)^{\frac{3}{2}}ax}{24}+\frac{\left(c\,x^2+a\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(c*x^2+a)^(5/2)+5/24*a*x*(c*x^2+a)^(3/2)+5/16*a^2*x*(c*x^2+a)^(1/2)+5
/16*a^3/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 1.38, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{cx^2+a}\,a^2x+\frac{5\,a^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*x + 5/24*(c*x^2 + a)^(3/2)*a*x + 5/16*sqrt(c*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

mupad [B] time = 0.20, size = 37, normalized size = 0.44

$$\frac{x\left(c\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{\left(\frac{c\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(5/2),x)
```

```
[Out] (x*(a + c*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
1)^(5/2)
```

sympy [A] time = 4.43, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24}+\frac{\sqrt{a}\,c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 + c*x**2/a)
/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*asinh(sqrt(c)*x/sqrt(
a))/(16*sqrt(c))
```

3.38 $\int \left(a+cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+cx^2\right)^{3/2} dx &= \frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+cx^2}\,dx\\
&= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx\\
&= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)\\
&= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.00

$$
\frac{1}{8}\sqrt{a+cx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+5ax+2cx^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(5*a*x + 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]
)/(Sqrt[c]*Sqrt[1 + (c*x^2)/a])))/8
```

IntegrateAlgebraic [A] time = 0.07, size = 60, normalized size = 0.92

$$
\frac{1}{8}\sqrt{a+cx^2}\left(5ax+2cx^3\right)-\frac{3a^2\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{8\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(5*a*x + 2*c*x^3))/8 - (3*a^2*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2]])/(8*Sqrt[c])
```

fricas [A] time = 0.41, size = 124, normalized size = 1.91

$$
\left[\frac{3\,a^2\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(2\,c^2x^3+5\,acx\right)\sqrt{cx^2+a}}{16\,c},-\frac{3\,a^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\left(2\,c^2x^3+5\,acx\right)\sqrt{cx^2+a}}{8\,c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2
*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c, -1/8*(3*a^2*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c]
```

giac [A] time = 0.43, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,cx^2+5\,a\right)\sqrt{cx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(c)*x + sqrt(c
*x^2 + a)))/sqrt(c)
```

maple [A] time = 0.04, size = 51, normalized size = 0.78

$$\frac{3a^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8\sqrt{c}}+\frac{3\sqrt{c\,x^2+a}\,ax}{8}+\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(c*x^2+a)^(3/2)+3/8*a*x*(c*x^2+a)^(1/2)+3/8*a^2/c^(1/2)*ln(c^(1/2)*x+
(c*x^2+a)^(1/2))
```

maxima [A] time = 1.30, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{cx^2+a}\,ax+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*x + 3/8*sqrt(c*x^2 + a)*a*x + 3/8*a^2*arcsinh(c*x/sqr
t(a*c))/sqrt(c)
```

mupad [B] time = 0.16, size = 37, normalized size = 0.57

$$\frac{x\left(c\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{\left(\frac{c\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2),x)
```

```
[Out] (x*(a + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
1)^(3/2)
```

sympy [A] time = 3.01, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{cx^2}{a}}}{8}+\frac{\sqrt{a}\,cx^3\sqrt{1+\frac{cx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**2/a)/4 + 3
*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))
```

3.39 $\int \sqrt{a+cx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a\int \frac{1}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)$$
$$= \frac{1}{2}x\sqrt{a+cx^2} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+cx^2} + \frac{a\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2],x]
[Out] (x*Sqrt[a + c*x^2])/2 + (a*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+cx^2} - \frac{a\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2],x]
[Out] (x*Sqrt[a + c*x^2])/2 - (a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

fricas [A] time = 0.42, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{cx^2+a}\,cx + a\sqrt{c}\,\log\left(-2\,cx^2 - 2\sqrt{cx^2+a}\,\sqrt{c}\,x - a\right)}{4\,c}, \frac{\sqrt{cx^2+a}\,cx - a\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^2 + a)*c*x + a*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a))/c, 1/2*(sqrt(c*x^2 + a)*c*x - a*sqrt(-c)*arctan(sqrt(-c)*x/sq
rt(c*x^2 + a)))/c]
```

giac [A] time = 0.40, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c
)
```

maple [A] time = 0.04, size = 36, normalized size = 0.78

$$\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}} + \frac{\sqrt{c\,x^2+a}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{cx^2+a}\,x + \frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x + 1/2*a*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

mupad [B] time = 0.13, size = 35, normalized size = 0.76

$$\frac{x\sqrt{c\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2),x)
```

```
[Out] (x*(a + c*x^2)^(1/2))/2 + (a*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/(2*c^(1/2)
)
```

sympy [A] time = 1.91, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + c*x**2/a)/2 + a*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c))
```

3.40 $\int \frac{1}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+cx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^2],x]
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + c*x^2],x]
[Out] -(Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/Sqrt[c])
```

fricas [A] time = 0.41, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)/sqrt(c), -sqrt(-c)*arc
tan(sqrt(-c)*x/sqrt(c*x^2 + a))/c]
```

giac [A] time = 0.41, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c)
```

maple [A] time = 0.04, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{c}\, x + \sqrt{c\, x^2 + a}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(1/2),x)
[Out] ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

maxima [A] time = 1.39, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

mupad [B] time = 0.19, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{c}\, x + \sqrt{c\, x^2 + a}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(1/2),x)
[Out] log(c^(1/2)*x + (a + c*x^2)^(1/2))/c^(1/2)
```

sympy [A] time = 1.05, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\, x}{\sqrt{a}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(1/2),x)
[Out] asinh(sqrt(c)*x/sqrt(a))/sqrt(c)
```

3.41 $\int \frac{1}{\left(a+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}} \, dx = \frac{x}{a\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

fricas [A] time = 0.40, size = 23, normalized size = 1.44

$$\frac{\sqrt{cx^2 + a}\,x}{acx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)
```

giac [A] time = 0.48, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2 + a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

maple [A] time = 0.04, size = 15, normalized size = 0.94

$$\frac{x}{\sqrt{c\,x^2 + a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2),x)
```

```
[Out] x/a/(c*x^2+a)^(1/2)
```

maxima [A] time = 1.25, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2 + a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + c*x^2)^(1/2))
```

sympy [A] time = 0.65, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

3.42 $\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx &= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\
&= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2cx^2\right)}{3a^2\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-5/2),x]
[Out] (x*(3*a + 2*c*x^2))/(3*a^2*(a + c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2cx^2\right)}{3a^2\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-5/2),x]
[Out] (x*(3*a + 2*c*x^2))/(3*a^2*(a + c*x^2)^(3/2))
```

fricas [A] time = 0.39, size = 47, normalized size = 1.21

$$\frac{\left(2\,cx^3+3\,ax\right)\sqrt{cx^2+a}}{3\left(a^2c^2x^4+2\,a^3cx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)
```

giac [A] time = 0.51, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,cx^2}{a^2}+\frac{3}{a}\right)}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)
```

maple [A] time = 0.04, size = 26, normalized size = 0.67

$$\frac{\left(2c\,x^{2}+3a\right)x}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*c*x^2+3*a)/(c*x^2+a)^(3/2)/a^2
```

maxima [A] time = 1.02, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{cx^{2}+a}\,a^{2}}+\frac{x}{3\left(cx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)
```

mupad [B] time = 0.19, size = 28, normalized size = 0.72

$$\frac{2\,x\,\left(c\,x^{2}+a\right)+a\,x}{3\,a^{2}\left(c\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + c*x^2) + a*x)/(3*a^2*(a + c*x^2)^(3/2))
```

sympy [B] time = 0.86, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}+\frac{2cx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a)
) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*
x**2/a))
```

3.43 $\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8x}{15a^3\sqrt{a+cx^2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{15a^3\sqrt{a+cx^2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-7/2),x]
```

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8*x)/(15*a^
3*Sqrt[a + c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx = \frac{x}{5a\left(a+cx^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}$$
$$= \frac{x}{5a\left(a+cx^2\right)^{5/2}} + \frac{4x}{15a^2\left(a+cx^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2}$$
$$= \frac{x}{5a\left(a+cx^2\right)^{5/2}} + \frac{4x}{15a^2\left(a+cx^2\right)^{3/2}} + \frac{8x}{15a^3\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{x\left(15a^2+20acx^2+8c^2x^4\right)}{15a^3\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-7/2),x]
[Out] (x*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4))/(15*a^3*(a + c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 40, normalized size = 0.69

$$\frac{x\left(15a^2+20acx^2+8c^2x^4\right)}{15a^3\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-7/2),x]
[Out] (x*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4))/(15*a^3*(a + c*x^2)^(5/2))
```

fricas [A] time = 0.40, size = 69, normalized size = 1.19

$$\frac{\left(8\,c^2x^5+20\,acx^3+15\,a^2x\right)\sqrt{cx^2+a}}{15\left(a^3c^3x^6+3\,a^4c^2x^4+3\,a^5cx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3*x^6 + 3*a
^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)
```

giac [A] time = 0.46, size = 41, normalized size = 0.71

$$\frac{\left(4\,x^2\left(\frac{2\,c^2x^2}{a^3} + \frac{5\,c}{a^2}\right) + \frac{15}{a}\right)x}{15\,\left(cx^2 + a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)
```

maple [A] time = 0.04, size = 37, normalized size = 0.64

$$\frac{\left(8c^2x^4 + 20ac\,x^2 + 15a^2\right)x}{15\left(c\,x^2 + a\right)^{\frac{5}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*x*(8*c^2*x^4+20*a*c*x^2+15*a^2)/(c*x^2+a)^(5/2)/a^3
```

maxima [A] time = 1.31, size = 46, normalized size = 0.79

$$\frac{8\,x}{15\,\sqrt{cx^2 + a}\,a^3} + \frac{4\,x}{15\,\left(cx^2 + a\right)^{\frac{3}{2}}a^2} + \frac{x}{5\,\left(cx^2 + a\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*x/((c*x
^2 + a)^(5/2)*a)
```

mupad [B] time = 0.19, size = 44, normalized size = 0.76

$$\frac{8\,x\,\left(c\,x^2 + a\right)^2 + 3\,a^2\,x + 4\,a\,x\,\left(c\,x^2 + a\right)}{15\,a^3\,\left(c\,x^2 + a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*x*(a + c*x^2)^2 + 3*a^2*x + 4*a*x*(a + c*x^2))/(15*a^3*(a + c*x^2)^(5/2)
)
```

sympy [B] time = 1.46, size = 413, normalized size = 7.12

$$\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(7/2),x)
```

```
[Out] 15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c
*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x*
*6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a) +
45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*
x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5/(1
5*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 4
5*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 +
c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/
2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))
```

3.44 $\int \frac{1}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6\int \frac{1}{(a+cx^2)^{7/2}}\,dx}{7a}$$

$$= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{24\int \frac{1}{(a+cx^2)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{16\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.66

$$\frac{x(35a^3+70a^2cx^2+56ac^2x^4+16c^3x^6)}{35a^4(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6))/(35*a^4*(a + c*x^2)
^(7/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 51, normalized size = 0.66

$$\frac{x(35a^3+70a^2cx^2+56ac^2x^4+16c^3x^6)}{35a^4(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6))/(35*a^4*(a + c*x^2)
^(7/2))
```

fricas [A] time = 0.41, size = 91, normalized size = 1.18

$$\frac{(16\,c^3x^7+56\,ac^2x^5+70\,a^2cx^3+35\,a^3x)\sqrt{cx^2+a}}{35\,(a^4c^4x^8+4\,a^5c^3x^6+6\,a^6c^2x^4+4\,a^7cx^2+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c*x^2 + a)/
(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*x^2 + a^8)
```

giac [A] time = 0.41, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4x^2\left(\frac{2c^3x^2}{a^4}+\frac{7c^2}{a^3}\right)+\frac{35c}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(cx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*c^3*x^2/a^4 + 7*c^2/a^3) + 35*c/a^2)*x^2 + 35/a)*x/(c*x^2
 + a)^(7/2)
```

maple [A] time = 0.05, size = 48, normalized size = 0.62

$$\frac{\left(16c^3x^6+56a\,c^2x^4+70a^2c\,x^2+35a^3\right)x}{35\left(c\,x^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*c^3*x^6+56*a*c^2*x^4+70*a^2*c*x^2+35*a^3)/(c*x^2+a)^(7/2)/a^4
```

maxima [A] time = 1.35, size = 61, normalized size = 0.79

$$\frac{16x}{35\sqrt{cx^2+a}\,a^4}+\frac{8x}{35\left(cx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6x}{35\left(cx^2+a\right)^{\frac{5}{2}}a^2}+\frac{x}{7\left(cx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(c*x^2 + a)*a^4) + 8/35*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*x/((c
*x^2 + a)^(5/2)*a^2) + 1/7*x/((c*x^2 + a)^(7/2)*a)
```

mupad [B] time = 0.20, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,a^4\,\sqrt{c\,x^2+a}}+\frac{8\,x}{35\,a^3\,{\left(c\,x^2+a\right)}^{3/2}}+\frac{6\,x}{35\,a^2\,{\left(c\,x^2+a\right)}^{5/2}}+\frac{x}{7\,a\,{\left(c\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(9/2),x)
```

```
[Out] (16*x)/(35*a^4*(a + c*x^2)^(1/2)) + (8*x)/(35*a^3*(a + c*x^2)^(3/2)) + (6*x
)/(35*a^2*(a + c*x^2)^(5/2)) + x/(7*a*(a + c*x^2)^(7/2))
```

sympy [B] time = 2.20, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 +
 c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**
3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 21
0*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1
+ c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c**3*
x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**
2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(
27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x*
*2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/
2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(
1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*
c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(1 +
 c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x
**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a
**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c
*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x**13
/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a)
 + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqr
t(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2
)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a
))
```

3.45 $\int \left(4+12x+9x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(3/2),x]
[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx=\frac{1}{12}(2+3x)\left(4+12x+9x^2\right)^{3/2}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 0.87

$$\frac{1}{12}(3x+2)\left((3x+2)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
[Out] ((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(4+12x+9x^2\right)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 12*x + 9*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(4 + 12*x + 9*x^2)^(3/2), x]
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{27}{4}\,x^4 + 18\,x^3 + 18\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

giac [B] time = 0.39, size = 45, normalized size = 1.96

$$\frac{3}{4}\left(3\,x^2 + 4\,x\right)^2 \mathrm{sgn}\left(3\,x + 2\right) + 2\left(3\,x^2 + 4\,x\right)\mathrm{sgn}\left(3\,x + 2\right) + \frac{4}{3}\,\mathrm{sgn}\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(3*x^2 + 4*x)^2*sgn(3*x + 2) + 2*(3*x^2 + 4*x)*sgn(3*x + 2) + 4/3*sgn(3
*x + 2)
```

maple [A] time = 0.05, size = 35, normalized size = 1.52

$$\frac{\left(27x^3 + 72x^2 + 72x + 32\right)\left((3x + 2)^2\right)^{\frac{3}{2}} x}{4\,(3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(3/2),x)
```

```
[Out] 1/4*x*(27*x^3+72*x^2+72*x+32)*((3*x+2)^2)^(3/2)/(3*x+2)^3
```

maxima [A] time = 2.96, size = 30, normalized size = 1.30

$$\frac{1}{4}\left(9\,x^2 + 12\,x + 4\right)^{\frac{3}{2}} x + \frac{1}{6}\left(9\,x^2 + 12\,x + 4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.83

$$\frac{(9\,x+6)\,\left(9\,x^2+12\,x+4\right)^{3/2}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 9*x^2 + 4)^(3/2),x)
[Out] ((9*x + 6)*(12*x + 9*x^2 + 4)^(3/2))/36
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(9x^2+12x+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(3/2),x)
[Out] Integral((9*x**2 + 12*x + 4)**(3/2), x)
```

3.46 $\int \sqrt{4+12x+9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4+12x+9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.09

$$\frac{x\sqrt{(3x+2)^2}\,(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] (x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{4+12x+9x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[4 + 12*x + 9*x^2], x]
```

fricas [A] time = 0.37, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2 + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 + 2*x
```

giac [A] time = 0.38, size = 26, normalized size = 1.13

$$\frac{1}{2}\left(3x^2 + 4x\right)\operatorname{sgn}(3x+2) + \frac{2}{3}\operatorname{sgn}(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)
```

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{(3x+4)\sqrt{(3x+2)^2}\,x}{6x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/2*x*(3*x+4)*((3*x+2)^2)^(1/2)/(3*x+2)
```

maxima [A] time = 2.86, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{9x^2 + 12x + 4}\,x + \frac{1}{3}\sqrt{9x^2 + 12x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.83

$$\frac{(3\,x+2)\,\sqrt{9\,x^2+12\,x+4}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 9*x^2 + 4)^(1/2),x)
```

```
[Out] ((3*x + 2)*(12*x + 9*x^2 + 4)^(1/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{9x^2+12x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 12*x + 4), x)
```

3.47 $\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx &= \frac{(6+9x)\int \frac{1}{6+9x}\,dx}{\sqrt{4+12x+9x^2}}\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 24, normalized size = 0.83

$$-\frac{1}{3}\log\left(\sqrt{9x^2+12x+4}-3x-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] -1/3*Log[-2 - 3*x + Sqrt[4 + 12*x + 9*x^2]]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

giac [A] time = 0.29, size = 25, normalized size = 0.86

$$\frac{\log\left(|3x+2|\left|\operatorname{sgn}\left(3x+2\right)\right|\right)}{3\operatorname{sgn}\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2)*abs(sgn(3*x + 2)))/sgn(3*x + 2)
```

maple [A] time = 0.05, size = 23, normalized size = 0.79

$$\frac{\left(3x+2\right)\ln\left(3x+2\right)}{3\sqrt{\left(3x+2\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/3*(3*x+2)*ln(3*x+2)/((3*x+2)^2)^(1/2)
```

maxima [A] time = 2.83, size = 6, normalized size = 0.21

$$\frac{1}{3} \log\left(x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 2/3)
```

mupad [B] time = 0.28, size = 14, normalized size = 0.48

$$\frac{\ln\left(9\,x+6\right)\,\mathrm{sign}\left(18\,x+12\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 9*x^2 + 4)^(1/2),x)
```

```
[Out] (log(9*x + 6)*sign(18*x + 12))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^2 + 12x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x + 4), x)
```

3.48 $\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {607}

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(-3/2),x]
```

```
[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{3x+2}{6\left((3x+2)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]
```

```
[Out] -1/6*(2 + 3*x)/((2 + 3*x)^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.18, size = 27, normalized size = 1.08

$$\frac{2}{3\left(-\sqrt{9x^2+12x+4}+3x+2\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(4 + 12*x + 9*x^2)^(-3/2),x]
[Out] 2/(3*(2 + 3*x - Sqrt[4 + 12*x + 9*x^2])^2)
```

fricas [A] time = 0.39, size = 14, normalized size = 0.56

$$-\frac{1}{6\left(9\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
[Out] -1/6/(9*x^2 + 12*x + 4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.05, size = 17, normalized size = 0.68

$$-\frac{3x+2}{6\left((3x+2)^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(3/2),x)
[Out] -1/6*(3*x+2)/((3*x+2)^2)^(3/2)
```

maxima [A] time = 2.89, size = 9, normalized size = 0.36

$$-\frac{1}{6\,(3\,x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/(3*x + 2)^2
```

mupad [B] time = 0.17, size = 21, normalized size = 0.84

$$-\frac{\sqrt{9\,x^2+12\,x+4}}{6\,{\left(3\,x+2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 9*x^2 + 4)^(3/2),x)
```

```
[Out] -(12*x + 9*x^2 + 4)^(1/2)/(6*(3*x + 2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(9x^2+12x+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

3.49 $\int \sqrt{4-12x+9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4-12x+9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.09

$$\frac{\sqrt{(2-3x)^2}\,x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] (Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{4-12x+9x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[4 - 12*x + 9*x^2],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[4 - 12*x + 9*x^2], x]
```

fricas [A] time = 0.38, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] 3/2*x^2 - 2*x
```

giac [A] time = 0.35, size = 26, normalized size = 1.13

$$\frac{1}{2}\left(3x^2 - 4x\right)\operatorname{sgn}(3x - 2) + \frac{2}{3}\operatorname{sgn}(3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)
```

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{(3x - 4)\sqrt{(3x - 2)^2}\,x}{6x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-2+3*x)^2)^(1/2),x)
[Out] 1/2*x*(3*x-4)*((-2+3*x)^2)^(1/2)/(-2+3*x)
```

maxima [A] time = 3.01, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{9x^2 - 12x + 4}\,x - \frac{1}{3}\sqrt{9x^2 - 12x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

mupad [B] time = 0.11, size = 13, normalized size = 0.57

$$\frac{|3\,x-2|\,\left(3\,x-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x - 2)^2)^(1/2),x)
[Out] (abs(3*x - 2)*(3*x - 2))/6
```

sympy [A] time = 0.08, size = 8, normalized size = 0.35

$$\frac{3x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)**2)**(1/2),x)
[Out] 3*x**2/2 - 2*x
```

3.50 $\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[4 - 12*x + 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx &= \frac{(-6+9x)\int \frac{1}{-6+9x}\,dx}{\sqrt{4-12x+9x^2}}\\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[(2 - 3*x)^2]
```

IntegrateAlgebraic [A] time = 0.08, size = 24, normalized size = 0.83

$$-\frac{1}{3}\log\left(\sqrt{9x^2-12x+4}-3x+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[4 - 12*x + 9*x^2],x]
[Out] -1/3*Log[2 - 3*x + Sqrt[4 - 12*x + 9*x^2]]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(3*x - 2)
```

giac [A] time = 0.46, size = 15, normalized size = 0.52

$$\frac{1}{3}\log\left(|3\,x-2|\right)\operatorname{sgn}\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*log(abs(3*x - 2))*sgn(3*x - 2)
```

maple [A] time = 0.05, size = 23, normalized size = 0.79

$$\frac{(3x-2)\ln\left(3x-2\right)}{3\sqrt{(3x-2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x-2)^2)^(1/2),x)
```

```
[Out] 1/3/((3*x-2)^2)^(1/2)*(3*x-2)*ln(3*x-2)
```

maxima [A] time = 2.97, size = 6, normalized size = 0.21

$$\frac{1}{3}\,\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x - 2/3)
```

mupad [B] time = 0.35, size = 14, normalized size = 0.48

$$\frac{\ln\left(3\,x-2\right)\,\mathrm{sign}\left(3\,x-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x - 2)^2)^(1/2),x)
```

```
[Out] (log(3*x - 2)*sign(3*x - 2))/3
```

sympy [A] time = 0.08, size = 7, normalized size = 0.24

$$\frac{\log\left(3x-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)**2)**(1/2),x)
```

```
[Out] log(3*x - 2)/3
```

3.51 $\int \sqrt{-4+12x-9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4+12x-9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.17

$$\frac{\sqrt{-(2-3x)^2}\,x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] (Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{-4+12x-9x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[-4 + 12*x - 9*x^2], x]
```

fricas [C] time = 0.38, size = 9, normalized size = 0.39

$$\frac{3}{2} i x^{2} - 2 i x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 - 2*I*x
```

giac [C] time = 0.38, size = 26, normalized size = 1.13

$$-\frac{1}{2} i \left(3 x^{2} - 4 x\right) \mathrm{sgn}\left(-3 x + 2\right) - \frac{2}{3} i \,\mathrm{sgn}\left(-3 x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)
```

maple [A] time = 0.05, size = 27, normalized size = 1.17

$$\frac{(3x-4)\sqrt{-(3x-2)^{2}}\, x}{6x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x-2)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x-4)*(-(3*x-2)^2)^(1/2)/(3*x-2)
```

maxima [A] time = 3.02, size = 30, normalized size = 1.30

$$\frac{1}{2} \sqrt{-9 x^{2} + 12 x - 4}\, x - \frac{1}{3} \sqrt{-9 x^{2} + 12 x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

mupad [B] time = 0.33, size = 18, normalized size = 0.78

$$\frac{(3\,x-2)\,\sqrt{-(3\,x-2)^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x - 2)^2)^(1/2),x)
```

```
[Out] ((3*x - 2)*(-(3*x - 2)^2)^(1/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(3x-2\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x - 2)**2), x)
```

3.52 $\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-4 + 12*x - 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx &= \frac{(6-9x)\int \frac{1}{6-9x}\,dx}{\sqrt{-4+12x-9x^2}}\\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.97

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-(2 - 3*x)^2]
```

IntegrateAlgebraic [C] time = 0.09, size = 30, normalized size = 1.03

$$\frac{1}{3}i\log\left(\sqrt{-9x^2+12x-4}-3ix+2i\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] (I/3)*Log[2*I - (3*I)*x + Sqrt[-4 + 12*x - 9*x^2]]
```

fricas [C] time = 0.38, size = 6, normalized size = 0.21

$$-\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*log(x - 2/3)
```

giac [C] time = 0.45, size = 23, normalized size = 0.79

$$\frac{i\log\left((-3ix+2i)\operatorname{sgn}(-3x+2)\right)}{3\operatorname{sgn}(-3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)
```

maple [A] time = 0.05, size = 25, normalized size = 0.86

$$\frac{(3x-2)\ln(3x-2)}{3\sqrt{-(3x-2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x-2)^2)^(1/2),x)
```

```
[Out] 1/3/(-(3*x-2)^2)^(1/2)*(3*x-2)*ln(3*x-2)
```

maxima [C] time = 2.90, size = 6, normalized size = 0.21

$$\frac{1}{3}i\,\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x - 2/3)
```

mupad [B] time = 0.25, size = 15, normalized size = 0.52

$$-\frac{\ln\left(2-3\,x\right)\,\mathrm{sign}\left(3\,x-2\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x - 2)^2)^(1/2),x)
```

```
[Out] -(log(2 - 3*x)*sign(3*x - 2)*1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\left(3x-2\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x - 2)**2), x)
```

3.53 $\int \sqrt{-4-12x-9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {609}

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4-12x-9x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.17

$$\frac{x\sqrt{-(3x+2)^2}\,(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] (x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{-4-12x-9x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[-4 - 12*x - 9*x^2],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[-4 - 12*x - 9*x^2], x]
```

fricas [C] time = 0.40, size = 9, normalized size = 0.39

$$\frac{3}{2} i x^2 + 2 i x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] 3/2*I*x^2 + 2*I*x
```

giac [C] time = 0.41, size = 26, normalized size = 1.13

$$-\frac{1}{2} i \left(3 x^2 + 4 x\right) \operatorname{sgn}\left(-3 x - 2\right) - \frac{2}{3} i \operatorname{sgn}\left(-3 x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)
```

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$\frac{(3x+4)\sqrt{-(3x+2)^2}\, x}{6x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x+2)^2)^(1/2),x)
[Out] 1/2*x*(3*x+4)*(-(3*x+2)^2)^(1/2)/(3*x+2)
```

maxima [A] time = 2.96, size = 30, normalized size = 1.30

$$\frac{1}{2} \sqrt{-9 x^2 - 12 x - 4}\, x + \frac{1}{3} \sqrt{-9 x^2 - 12 x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

mupad [B] time = 0.07, size = 18, normalized size = 0.78

$$\frac{(3\,x+2)\,\sqrt{-(3\,x+2)^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x + 2)^2)^(1/2),x)
```

```
[Out] ((3*x + 2)*(-(3*x + 2)^2)^(1/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x + 2)**2), x)
```

3.54 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx &= -\left(-\frac{(-6-9x)\int \frac{1}{-6-9x}\,dx}{\sqrt{-4-12x-9x^2}}\right)\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 0.97

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

IntegrateAlgebraic [C] time = 0.08, size = 30, normalized size = 1.03

$$\frac{1}{3}i\log\left(\sqrt{-9x^2-12x-4}-3ix-2i\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] (I/3)*Log[-2*I - (3*I)*x + Sqrt[-4 - 12*x - 9*x^2]]
```

fricas [C] time = 0.38, size = 6, normalized size = 0.21

$$-\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*I*log(x + 2/3)
```

giac [C] time = 0.34, size = 23, normalized size = 0.79

$$\frac{i\log\left((-3i\,x-2i)\mathrm{sgn}\left(-3\,x-2\right)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

maple [A] time = 0.05, size = 25, normalized size = 0.86

$$\frac{(3x+2)\ln\left(3x+2\right)}{3\sqrt{-\left(3x+2\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x+2)^2)^(1/2),x)
```

```
[Out] 1/3*(3*x+2)/(-(3*x+2)^2)^(1/2)*ln(3*x+2)
```

maxima [C] time = 2.78, size = 6, normalized size = 0.21

$$\frac{1}{3} i \log\left(x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*I*log(x + 2/3)
```

mupad [B] time = 0.29, size = 15, normalized size = 0.52

$$-\frac{\ln\left(-3\,x-2\right)\,\mathrm{sign}\left(3\,x+2\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x + 2)^2)^(1/2),x)
[Out] -(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(3x + 2\right)^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-(3*x + 2)**2), x)
```

3.55 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (5*(1 - b -
 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6) + (1 - b - 2*c*x)^
10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-1+b)+cx\right)^5 \left(\frac{1+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(\left(\frac{1}{2}(-1+b)+cx\right)^5+5\left(\frac{1}{2}(-1+b)+cx\right)^6+10\left(\frac{1}{2}(-1+b)+cx\right)^7+10\left(\frac{1}{2}(-1+\right.\right.}{c^5}$$
$$= \frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6}-\frac{5(1-b-2cx)^9}{2304c^6}+\frac{(1-b-2}{2048c}$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.90

$$\frac{5}{8}\left(3b^3-b\right)c^2x^8+\frac{\left(b^2-1\right)^5x}{1024c^5}+\frac{5b\left(b^2-1\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-1\right)c^3x^9+\frac{5\left(b^2-1\right)^3\left(9b^2-1\right)x^3}{768c^3}+\frac{5b\left(b^2-1\right)^2\left(3b^2-1\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-14b^2+1\right)cx^7+\frac{\left(b^2-1\right)\left(21b^4-14b^2+1\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-70b^2+15\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 + b
^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(64
*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2 + 6
3*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*(-b + 3*b^3)*c^2*x
^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[((-1 + b^2)/(4*c) + b*x + c*x^2)^5, x]
```

fricas [B] time = 0.38, size = 233, normalized size = 2.14

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\left(9\,b^2-1\right)c^8x^9+443520\left(3\,b^3-b\right)c^7x^8+63360\left(21\,b^4-14\,b^2+1\right)c^6x^7+14784\left(63\,b^5-70\,b^3+15\,b\right)c^5x^6+22176\left(21\,b^6-35\,b^4+15\,b^2-1\right)c^4x^5+55440\left(3\,b^7-7\,b^5+5\,b^3-b\right)c^3x^4+4620\left(9\,b^8-28\,b^6+30\,b^4-12\,b^2+1\right)c^2x^3+6930\left(b^9-4\,b^7+6\,b^5-4\,b^3+b\right)cx^2+693\left(b^{10}-5\,b^8+10\,b^6-10\,b^4+5\,b^2-1\right)x}{709632\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9 +
 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784*(
63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c^4*
x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6 + 30
*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*x^2 +
 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

giac [B] time = 0.43, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 98560\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 443520\,bc^{7}x^{8} + 465696\,b^{6}c^{4}x^{5} - 887040\,b^{2}c^{6}x^{7} + 166320\,b^{7}c^{3}x^{4} - 1034880\,b^{3}c^{5}x^{6} + 41580\,b^{8}c^{2}x^{3} - 776160\,b^{4}c^{4}x^{5} + 63360\,c^{6}x^{7} + 6930\,b^{9}cx^{2} - 388080\,b^{5}c^{3}x^{4} + 221760\,bc^{5}x^{6} + 693\,b^{10}x - 129360\,b^{6}c^{2}x^{3} + 332640\,b^{2}c^{4}x^{5} - 27720\,b^{7}cx^{2} + 277200\,b^{3}c^{3}x^{4} - 3465\,b^{8}x + 138600\,b^{4}c^{2}x^{3} - 22176\,c^{4}x^{5} + 41580\,b^{5}cx^{2} - 55440\,bc^{3}x^{4} + 6930\,b^{6}x - 55440\,b^{2}c^{2}x^{3} - 27720\,b^{3}cx^{2} - 6930\,b^{4}x + 4620\,c^{2}x^{3} + 6930\,bcx^{2} + 3465\,b^{2}x - 693\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6 -
443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*c^3
*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 63360
*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 693*b^1
0*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277200*b^
3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*b^5*c*x
^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c*x^2 - 6
930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5
```

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^{5}x^{11}}{11}+\frac{b\,c^{4}x^{10}}{2}+\frac{\left(4b^{2}c^{3}+\frac{\left(b^{2}-1\right)c^{3}}{4}+\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)c^{2}\right)c\right)x^{9}}{9}+\frac{\left(\left(b^{2}-1\right)b\,c^{2}+\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)c^{2}\right)b+\left(\left(b^{2}-1\right)bc+4\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)bc\right)c\right)x^{8}}{8}+\frac{\left(\left(\left(b^{2}-1\right)bc+4\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)bc\right)b+\left(2\left(b^{2}-1\right)b^{2}+\frac{\left(b^{2}-1\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)^{2}\right)c+\frac{\left(b^{2}-1\right)\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)c^{2}\right)}{4c}\right)x^{7}}{7}+\frac{\left(\left(2\left(b^{2}-1\right)b^{2}+\frac{\left(b^{2}-1\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)^{2}\right)b+\left(\frac{\left(b^{2}-1\right)^{2}b}{4c}+\frac{\left(b^{2}-1\right)\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)b}{c}\right)c+\frac{\left(b^{2}-1\right)\left(\left(b^{2}-1\right)bc+4\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)bc\right)}{4c}\right)x^{6}}{6}+\frac{\left(\left(\frac{\left(b^{2}-1\right)^{2}b}{4c}+\frac{\left(b^{2}-1\right)\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)b}{c}\right)b+\left(\frac{\left(b^{2}-1\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-1\right)^{2}\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)}{8c^{2}}\right)c+\frac{\left(b^{2}-1\right)\left(2\left(b^{2}-1\right)b^{2}+\frac{\left(b^{2}-1\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)^{2}\right)}{4c}\right)x^{5}}{5}+\frac{\left(\left(\frac{\left(b^{2}-1\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-1\right)^{2}\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)}{8c^{2}}\right)b+\frac{\left(b^{2}-1\right)^{3}b}{16c^{2}}+\frac{\left(b^{2}-1\right)\left(\frac{\left(b^{2}-1\right)^{2}b}{4c}+\frac{\left(b^{2}-1\right)\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)b}{c}\right)}{4c}\right)x^{4}}{4}+\frac{\left(\frac{\left(b^{2}-1\right)^{3}b^{2}}{16c^{3}}+\frac{\left(b^{2}-1\right)\left(\frac{\left(b^{2}-1\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-1\right)^{2}\left(\frac{3b^{2}}{2}-\frac{1}{2}\right)}{8c^{2}}\right)}{4c}+\frac{\left(b^{2}-1\right)^{4}}{256c^{3}}\right)x^{3}}{3}+\frac{5\left(b^{2}-1\right)^{4}b\,x^{2}}{512c^{4}}+\frac{\left(b^{2}-1\right)^{5}x}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-1)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-1)*c^2*b+b*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2
)+c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c))*x^8+1/7*(1/4*(b^2-1)/c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2)+b*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+c*(1/8*(b^2-1)^2+2*(
b^2-1)*b^2+(3/2*b^2-1/2)^2))*x^7+1/6*(1/4*(b^2-1)/c*((b^2-1)*c*b+4*(3/2*b^2
-1/2)*b*c)+b*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+c*(1/4*(b^2-1)^2
/c*b+(b^2-1)/c*b*(3/2*b^2-1/2)))*x^6+1/5*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2+2*(b
^2-1)*b^2+(3/2*b^2-1/2)^2)+b*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+
c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-1)/c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+b*(1/8*(b^2-1)^2/c^2*(
3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16/c^2*(b^2-1)^3*b)*x^4+1/3*(1/4*(b^2
-1)/c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16*b^2*(b^2
-1)^3/c^3+1/256/c^3*(b^2-1)^4)*x^3+5/512*(b^2-1)^4/c^4*b*x^2+1/1024*(b^2-1)
^5/c^5*x
```

maxima [B] time = 1.41, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5\left(2\,cx^3 + 3\,bx^2\right)\left(b^2-1\right)^4}{1536\,c^4} + \frac{\left(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x^3\right)\left(b^2-1\right)^3}{192\,c^3} + \frac{\left(20\,c^3x^7 + 70\,bc^2x^6 + 84\,b^2cx^5 + 35\,b^3x^4\right)\left(b^2-1\right)^2}{224\,c^2} + \frac{\left(70\,c^4x^9 + 315\,bc^3x^8 + 540\,b^2c^2x^7 + 420\,b^3cx^6 + 126\,b^4x^5\right)\left(b^2-1\right)}{504\,c} + \frac{\left(b^2-1\right)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 1)/c + 1/1024*(b^2 - 1)^5*x/c^5
```

mupad [B] time = 0.31, size = 184, normalized size = 1.69

$$\frac{c^5\,x^{11}}{11} + \frac{x\,\left(b^2-1\right)^5}{1024\,c^5} + \frac{b\,x^6\,\left(63\,b^4 - 70\,b^2 + 15\right)}{48} + \frac{5\,c\,x^7\,\left(21\,b^4 - 14\,b^2 + 1\right)}{56} + \frac{b\,c^4\,x^{10}}{2} + \frac{5\,c^3\,x^9\,\left(9\,b^2-1\right)}{36} + \frac{x^5\,\left(21\,b^6 - 35\,b^4 + 15\,b^2 - 1\right)}{32\,c} + \frac{5\,b\,c^2\,x^8\,\left(3\,b^2-1\right)}{8} + \frac{5\,b\,x^2\,\left(b^2-1\right)^4}{512\,c^4} + \frac{5\,x^3\,\left(b^2-1\right)^3\,\left(9\,b^2-1\right)}{768\,c^3} + \frac{5\,b\,x^4\,\left(b^2-1\right)^2\,\left(3\,b^2-1\right)}{64\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 1/4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 1)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 70*b^2 + 15))
/48 + (5*c*x^7*(21*b^4 - 14*b^2 + 1))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9*b
^2 - 1))/36 + (x^5*(15*b^2 - 35*b^4 + 21*b^6 - 1))/(32*c) + (5*b*c^2*x^8*(3
*b^2 - 1))/8 + (5*b*x^2*(b^2 - 1)^4)/(512*c^4) + (5*x^3*(b^2 - 1)^3*(9*b^2
- 1))/(768*c^3) + (5*b*x^4*(b^2 - 1)^2*(3*b^2 - 1))/(64*c^2)
```

sympy [B] time = 0.19, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{36}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{5bc^2}{8}\right) + x^7\left(\frac{15b^4c}{8} - \frac{5b^2c}{4} + \frac{5c}{56}\right) + x^6\left(\frac{21b^5}{16} - \frac{35b^3}{24} + \frac{5b}{16}\right) + \frac{x^5\left(21b^6 - 35b^4 + 15b^2 - 1\right)}{32c} + \frac{x^4\left(15b^7 - 35b^5 + 25b^3 - 5b\right)}{64c^2} + \frac{x^3\left(45b^8 - 140b^6 + 150b^4 - 60b^2 + 5\right)}{768c^3} + \frac{x^2\left(5b^9 - 20b^7 + 30b^5 - 20b^3 + 5b\right)}{512c^4} + \frac{x\left(b^{10} - 5b^8 + 10b^6 - 10b^4 + 5b^2 - 1\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(1
5*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) + x*
*6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**2 -
 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*(45*
b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 - 20*b*
*7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**6 - 10
*b**4 + 5*b**2 - 1)/(1024*c**5)
```

3.56 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*(2 - b - 2
*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) + (2 - b - 2*c*x)^10/(
1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-2+b)+cx\right)^5 \left(\frac{2+b}{2}+cx\right)^5 dx}{c^5}$$

$$= \frac{\int \left(32\left(\frac{1}{2}(-2+b)+cx\right)^5+80\left(\frac{1}{2}(-2+b)+cx\right)^6+80\left(\frac{1}{2}(-2+b)+cx\right)^7+40\left(\frac{1}{2}(\cdots\right.\right.}{c^5}$$

$$= \frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6}-\frac{5(2-b-2cx)^9}{576c^6}+\frac{(2-b-\cdots}{1024\cdots}$$

Mathematica [A] time = 0.04, size = 207, normalized size = 1.90

$$\frac{5}{8}\left(3b^3-4b\right)c^2x^8+\frac{\left(b^2-4\right)^5x}{1024c^5}+\frac{5b\left(b^2-4\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-4\right)c^3x^9+\frac{5\left(b^2-4\right)^3\left(9b^2-4\right)x^3}{768c^3}+\frac{5b\left(b^2-4\right)^2\left(3b^2-4\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-56b^2+16\right)cx^7+\frac{\left(b^2-4\right)\left(21b^4-56b^2+16\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-280b^2+240\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 + b
^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(64
*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b^2
+ 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*(-4*b + 3*b^3)
*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[((-4 + b^2)/(4*c) + b*x + c*x^2)^5, x]
```

fricas [B] time = 0.39, size = 235, normalized size = 2.16

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\,(9\,b^2-4)c^8x^9+443520\,(3\,b^3-4\,b)c^7x^8+63360\,(21\,b^4-56\,b^2+16)c^6x^7+14784\,(63\,b^5-280\,b^3+240\,b)c^5x^6+22176\,(21\,b^6-140\,b^4+240\,b^2-64)c^4x^5+55440\,(3\,b^7-28\,b^5+80\,b^3-64\,b)c^3x^4+4620\,(9\,b^8-112\,b^6+480\,b^4-768\,b^2+256)c^2x^3+6930\,(b^9-16\,b^7+96\,b^5-256\,b^3+256\,b)cx^2+693\,(b^{10}-20\,b^8+160\,b^6-640\,b^4+1280\,b^2-1024)x}{709632\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9 +
 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 1478
4*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2 -
64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*b^8
- 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96*b^5
- 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1280*b^
2 - 1024)*x)/c^5
```

giac [**B**] time = 0.41, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 394240\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 1774080\,bc^{7}x^{8} + 465696\,b^{6}c^{4}x^{5} - 3548160\,b^{2}c^{6}x^{7} + 166320\,b^{7}c^{3}x^{4} - 4139520\,b^{3}c^{5}x^{6} + 41580\,b^{8}c^{2}x^{3} - 3104640\,b^{4}c^{4}x^{5} + 1013760\,c^{6}x^{7} + 6930\,b^{9}cx^{2} - 1552320\,b^{5}c^{3}x^{4} + 3548160\,bc^{5}x^{6} + 693\,b^{10}x - 517440\,b^{6}c^{2}x^{3} + 5322240\,b^{2}c^{4}x^{5} - 110880\,b^{7}cx^{2} + 4435200\,b^{3}c^{3}x^{4} - 13860\,b^{8}x + 2217600\,b^{4}c^{2}x^{3} - 1419264\,c^{4}x^{5} + 665280\,b^{5}cx^{2} - 3548160\,bc^{3}x^{4} + 110880\,b^{6}x - 3548160\,b^{2}c^{2}x^{3} - 1774080\,b^{3}cx^{2} - 443520\,b^{4}x + 1182720\,c^{2}x^{3} + 1774080\,bcx^{2} + 887040\,b^{2}x - 709632\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6 -
 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b^7*
c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5 + 1
013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b*c^5*x^6 +
 693*b^10*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c*x^2 +
 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c^4*x^5
+ 665280*b^5*c*x^2 - 3548160*b*c^3*x^4 + 110880*b^6*x - 3548160*b^2*c^2*x^3
 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b*c*x^2 + 8
87040*b^2*x - 709632*x)/c^5
```

maple [**B**] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^{5}x^{11}}{11} + \frac{b\,c^{4}x^{10}}{2} + \frac{\left(4b^{2}c^{3} + \frac{\left(b^{2}-4\right)c^{3}}{4} + \left(4b^{2}c^{2} + 2\left(\frac{3b^{2}}{2}-2\right)c^{2}\right)c\right)x^{9}}{9} + \frac{\left(\left(b^{2}-4\right)b\,c^{2} + \left(4b^{2}c^{2} + 2\left(\frac{3b^{2}}{2}-2\right)c^{2}\right)b + \left(\left(b^{2}-4\right)bc + 4\left(\frac{3b^{2}}{2}-2\right)bc\right)c\right)x^{8}}{8} + \frac{\left(\left(\left(b^{2}-4\right)bc + 4\left(\frac{3b^{2}}{2}-2\right)bc\right)b + \left(2\left(b^{2}-4\right)b^{2} + \frac{\left(b^{2}-4\right)^{2}}{8} + \left(\frac{3b^{2}}{2}-2\right)^{2}\right)c + \frac{\left(b^{2}-4\right)\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-2\right)c^{2}\right)}{4c}\right)x^{7}}{7} + \frac{\left(\left(2\left(b^{2}-4\right)b^{2} + \frac{\left(b^{2}-4\right)^{2}}{8} + \left(\frac{3b^{2}}{2}-2\right)^{2}\right)b + \left(\frac{\left(b^{2}-4\right)^{2}b}{4c} + \frac{\left(b^{2}-4\right)\left(\frac{3b^{2}}{2}-2\right)b}{c}\right)c + \frac{\left(b^{2}-4\right)\left(\left(b^{2}-4\right)bc+4\left(\frac{3b^{2}}{2}-2\right)bc\right)}{4c}\right)x^{6}}{6} + \frac{\left(\left(\frac{\left(b^{2}-4\right)^{2}b}{4c} + \frac{\left(b^{2}-4\right)\left(\frac{3b^{2}}{2}-2\right)b}{c}\right)b + \left(\frac{\left(b^{2}-4\right)^{2}b^{2}}{4c^{2}} + \frac{\left(b^{2}-4\right)^{2}\left(\frac{3b^{2}}{2}-2\right)}{8c^{2}}\right)c + \frac{\left(b^{2}-4\right)\left(2\left(b^{2}-4\right)b^{2}+\frac{\left(b^{2}-4\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-2\right)^{2}\right)}{4c}\right)x^{5}}{5} + \frac{\left(\left(\frac{\left(b^{2}-4\right)^{2}b^{2}}{4c^{2}} + \frac{\left(b^{2}-4\right)^{2}\left(\frac{3b^{2}}{2}-2\right)}{8c^{2}}\right)b + \frac{\left(b^{2}-4\right)^{3}b}{16c^{2}} + \frac{\left(b^{2}-4\right)\left(\frac{\left(b^{2}-4\right)^{2}b}{4c}+\frac{\left(b^{2}-4\right)\left(\frac{3b^{2}}{2}-2\right)b}{c}\right)}{4c}\right)x^{4}}{4} + \frac{\left(\frac{\left(b^{2}-4\right)^{3}b^{2}}{16c^{3}} + \frac{\left(b^{2}-4\right)\left(\frac{\left(b^{2}-4\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-4\right)^{2}\left(\frac{3b^{2}}{2}-2\right)}{8c^{2}}\right)}{4c} + \frac{\left(b^{2}-4\right)^{4}}{256c^{3}}\right)x^{3}}{3} + \frac{5\left(b^{2}-4\right)^{4}b\,x^{2}}{512c^{4}} + \frac{\left(b^{2}-4\right)^{5}x}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-4)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-2
)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-4)*c^2*b+b*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+c*
((b^2-4)*c*b+4*(3/2*b^2-2)*b*c))*x^8+1/7*(1/4*(b^2-4)/c*(2*(3/2*b^2-2)*c^2+
4*b^2*c^2)+b*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2
+(3/2*b^2-2)^2))*x^7+1/6*(1/4*(b^2-4)/c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+b*(
1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2)+c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b
*(3/2*b^2-2)))*x^6+1/5*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2
-2)^2)+b*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+c*(1/8*(b^2-4)^2/c^2*(
3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-4)/c*(1/4*(b^2-4)^2/c*
b+(b^2-4)/c*b*(3/2*b^2-2))+b*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c
^2*b^2)+1/16/c^2*(b^2-4)^3*b)*x^4+1/3*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2/c^2*(3/
```

```
2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16*b^2*(b^2-4)^3/c^3+1/256/c^3*(b^2-4)^4)
*x^3+5/512*(b^2-4)^4/c^4*b*x^2+1/1024*(b^2-4)^5/c^5*x
```

maxima [B] time = 1.52, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6+\frac{5\left(2cx^3+3bx^2\right)\left(b^2-4\right)^4}{1536c^4}+\frac{\left(6c^2x^5+15bcx^4+10b^2x^3\right)\left(b^2-4\right)^3}{192c^3}+\frac{\left(20c^3x^7+70bc^2x^6+84b^2cx^5+35b^3x^4\right)\left(b^2-4\right)^2}{224c^2}+\frac{\left(70c^4x^9+315bc^3x^8+540b^2c^2x^7+420b^3cx^6+126b^4x^5\right)\left(b^2-4\right)}{504c}+\frac{\left(b^2-4\right)^5x}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 4)/c + 1/1024*(b^2 - 4)^5*x/c^5
```

mupad [B] time = 0.33, size = 184, normalized size = 1.69

$$\frac{c^5x^{11}}{11}+\frac{x\left(b^2-4\right)^5}{1024c^5}+\frac{bx^6\left(63b^4-280b^2+240\right)}{48}+\frac{5cx^7\left(21b^4-56b^2+16\right)}{56}+\frac{bc^4x^{10}}{2}+\frac{5c^3x^9\left(9b^2-4\right)}{36}+\frac{x^5\left(21b^6-140b^4+240b^2-64\right)}{32c}+\frac{5bc^2x^8\left(3b^2-4\right)}{8}+\frac{5bx^2\left(b^2-4\right)^4}{512c^4}+\frac{5x^3\left(b^2-4\right)^3\left(9b^2-4\right)}{768c^3}+\frac{5bx^4\left(b^2-4\right)^2\left(3b^2-4\right)}{64c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 1)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 4)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 280*b^2 + 240
))/48 + (5*c*x^7*(21*b^4 - 56*b^2 + 16))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(
9*b^2 - 4))/36 + (x^5*(240*b^2 - 140*b^4 + 21*b^6 - 64))/(32*c) + (5*b*c^2*
x^8*(3*b^2 - 4))/8 + (5*b*x^2*(b^2 - 4)^4)/(512*c^4) + (5*x^3*(b^2 - 4)^3*(
9*b^2 - 4))/(768*c^3) + (5*b*x^4*(b^2 - 4)^2*(3*b^2 - 4))/(64*c^2)
```

sympy [B] time = 0.20, size = 250, normalized size = 2.29

$$\frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\left(\frac{5b^2c^3}{4}-\frac{5c^3}{9}\right)+x^8\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{2}\right)+x^7\left(\frac{15b^4c}{8}-5b^2c+\frac{10c}{7}\right)+x^6\left(\frac{21b^5}{16}-\frac{35b^3}{6}+5b\right)+\frac{x^5\left(21b^6-140b^4+240b^2-64\right)}{32c}+\frac{x^4\left(15b^7-140b^5+400b^3-320b\right)}{64c^2}+\frac{x^3\left(45b^8-560b^6+2400b^4-3840b^2+1280\right)}{768c^3}+\frac{x^2\left(5b^9-80b^7+480b^5-1280b^3+1280b\right)}{512c^4}+\frac{x\left(b^{10}-20b^8+160b^6-640b^4+1280b^2-1024\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(15
*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**6*
(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 64)/
(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*(45*
b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b**9 -
 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*b**8 +
 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

3.57 $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+3)^{11}}{22528c^6}+\frac{3(-b-2cx+3)^{10}}{2048c^6}-\frac{5(-b-2cx+3)^9}{256c^6}+\frac{135(-b-2cx+3)^8}{1024c^6}-\frac{405(-b-2cx+3)^7}{896c^6}+\frac{81(-b-2}{128}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+3)^{11}}{22528c^6}+\frac{3(-b-2cx+3)^{10}}{2048c^6}-\frac{5(-b-2cx+3)^9}{256c^6}+\frac{135(-b-2cx+3)^8}{1024c^6}-\frac{405(-b-2cx+3)^7}{896c^6}+\frac{81(-b-2cx+3)^6}{128c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c^6) + (135
*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/(256*c^6) + (3*(3 -
b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^11/(22528*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-3+b)+cx\right)^5 \left(\frac{3+b}{2}+cx\right)^5 dx}{c^5}$$

$$= \frac{\int \left(243\left(\frac{1}{2}(-3+b)+cx\right)^5 + 405\left(\frac{1}{2}(-3+b)+cx\right)^6 + 270\left(\frac{1}{2}(-3+b)+cx\right)^7 + 90\right.}{c^5}$$

$$= \frac{81(3-b-2cx)^6}{128c^6} - \frac{405(3-b-2cx)^7}{896c^6} + \frac{135(3-b-2cx)^8}{1024c^6} - \frac{5(3-b-2cx)^9}{256c^6} +$$

Mathematica [A] time = 0.03, size = 199, normalized size = 1.83

$$\frac{15}{8}\left(b^3-3b\right)c^2x^8+\frac{\left(b^2-9\right)^5x}{1024c^5}+\frac{5b\left(b^2-9\right)^4x^2}{512c^4}+\frac{5}{4}\left(b^2-1\right)c^3x^9+\frac{15\left(b^2-9\right)^3\left(b^2-1\right)x^3}{256c^3}+\frac{15b\left(b^2-9\right)^2\left(b^2-3\right)x^4}{64c^2}+\frac{15}{56}\left(7b^4-42b^2+27\right)cx^7+\frac{3\left(b^2-9\right)\left(7b^4-42b^2+27\right)x^5}{32c}+\frac{3}{16}b\left(7b^4-70b^2+135\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 +
b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64*c
^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b^2
+ 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*(-3*b + b^3)*c
^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[((-9 + b^2)/(4*c) + b*x + c*x^2)^5, x]
```

fricas [B] time = 0.39, size = 227, normalized size = 2.08

$$\frac{7168\,c^{10}x^{11}+39424\,bc^9x^{10}+98560\left(b^2-1\right)c^8x^9+147840\left(b^3-3\,b\right)c^7x^8+21120\left(7\,b^4-42\,b^2+27\right)c^6x^7+14784\left(7\,b^5-70\,b^3+135\,b\right)c^5x^6+7392\left(7\,b^6-105\,b^4+405\,b^2-243\right)c^4x^5+18480\left(b^7-21\,b^5+135\,b^3-243\,b\right)c^3x^4+4620\left(b^8-28\,b^6+270\,b^4-972\,b^2+729\right)c^2x^3+770\left(b^9-36\,b^7+486\,b^5-2916\,b^3+6561\,b\right)cx^2+77\left(b^{10}-45\,b^8+810\,b^6-7290\,b^4+32805\,b^2-59049\right)x}{78848\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 1478
40*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b^5
 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*x^5
 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6 + 27
0*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^3 + 6
561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 59049)*
x)/c^5
```

giac [B] time = 0.45, size = 334, normalized size = 3.06

$$\frac{7168 c^{10} x^{11} + 39424 b c^{9} x^{10} + 98560 b^{2} c^{8} x^{9} + 147840 b^{3} c^{7} x^{8} + 147840 b^{4} c^{6} x^{7} - 98560 c^{8} x^{9} + 103488 b^{5} c^{5} x^{6} - 443520 b c^{7} x^{8} + 51744 b^{6} c^{4} x^{5} - 887040 b^{2} c^{6} x^{7} + 18480 b^{7} c^{3} x^{4} - 1034880 b^{3} c^{5} x^{6} + 4620 b^{8} c^{2} x^{3} - 776160 b^{4} c^{4} x^{5} + 570240 c^{6} x^{7} + 770 b^{9} c x^{2} - 388080 b^{5} c^{3} x^{4} + 1995840 b c^{5} x^{6} + 77 b^{10} x - 129360 b^{6} c^{2} x^{3} + 2993760 b^{2} c^{4} x^{5} - 27720 b^{7} c x^{2} + 2494800 b^{3} c^{3} x^{4} - 3465 b^{8} x + 1247400 b^{4} c^{2} x^{3} - 1796256 c^{4} x^{5} + 374220 b^{5} c x^{2} - 4490640 b c^{3} x^{4} + 62370 b^{6} x - 4490640 b^{2} c^{2} x^{3} - 2245320 b^{3} c x^{2} - 561330 b^{4} x + 3367980 c^{2} x^{3} + 5051970 b c x^{2} + 2525985 b^{2} x - 4546773 x}{78848 c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^3
*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 443520
*b*c^7*x^8 + 51744*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 18480*b^7*c^3*x^4 - 1
034880*b^3*c^5*x^6 + 4620*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 570240*c^6*x^7
 + 770*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 1995840*b*c^5*x^6 + 77*b^10*x - 129
360*b^6*c^2*x^3 + 2993760*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 2494800*b^3*c^3*x
^4 - 3465*b^8*x + 1247400*b^4*c^2*x^3 - 1796256*c^4*x^5 + 374220*b^5*c*x^2
- 4490640*b*c^3*x^4 + 62370*b^6*x - 4490640*b^2*c^2*x^3 - 2245320*b^3*c*x^2
 - 561330*b^4*x + 3367980*c^2*x^3 + 5051970*b*c*x^2 + 2525985*b^2*x - 45467
73*x)/c^5
```

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^{5} x^{11}}{11}+\frac{b c^{4} x^{10}}{2}+\frac{\left(4 b^{2} c^{3}+\frac{\left(b^{2}-9\right) c^{3}}{4}+\left(4 b^{2} c^{2}+2\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) c^{2}\right) c\right) x^{9}}{9}+\frac{\left(\left(b^{2}-9\right) b c^{2}+\left(4 b^{2} c^{2}+2\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) c^{2}\right) b+\left(\left(b^{2}-9\right) b c+4\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b c\right) c\right) x^{8}}{8}+\frac{\left(\left(\left(b^{2}-9\right) b c+4\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b c\right) b+\left(2\left(b^{2}-9\right) b^{2}+\frac{\left(b^{2}-9\right)^{2}}{8}+\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)^{2}\right) c+\frac{\left(b^{2}-9\right)\left(4 b^{2} c^{2}+2\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) c^{2}\right)}{4 c}\right) x^{7}}{7}+\frac{\left(\left(2\left(b^{2}-9\right) b^{2}+\frac{\left(b^{2}-9\right)^{2}}{8}+\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)^{2}\right) b+\left(\frac{\left(b^{2}-9\right)^{2} b}{4 c}+\frac{\left(b^{2}-9\right)\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b}{c}\right) c+\frac{\left(b^{2}-9\right)\left(\left(b^{2}-9\right) b c+4\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b c\right)}{4 c}\right) x^{6}}{6}+\frac{\left(\left(\frac{\left(b^{2}-9\right)^{2} b}{4 c}+\frac{\left(b^{2}-9\right)\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b}{c}\right) b+\left(\frac{\left(b^{2}-9\right)^{2} b^{2}}{4 c^{2}}+\frac{\left(b^{2}-9\right)^{2}\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)}{8 c^{2}}\right) c+\frac{\left(b^{2}-9\right)\left(2\left(b^{2}-9\right) b^{2}+\frac{\left(b^{2}-9\right)^{2}}{8}+\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)^{2}\right)}{4 c}\right) x^{5}}{5}+\frac{\left(\left(\frac{\left(b^{2}-9\right)^{2} b^{2}}{4 c^{2}}+\frac{\left(b^{2}-9\right)^{2}\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)}{8 c^{2}}\right) b+\frac{\left(b^{2}-9\right)^{3} b}{16 c^{2}}+\frac{\left(b^{2}-9\right)\left(\frac{\left(b^{2}-9\right)^{2} b}{4 c}+\frac{\left(b^{2}-9\right)\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right) b}{c}\right)}{4 c}\right) x^{4}}{4}+\frac{\left(\frac{\left(b^{2}-9\right)^{3} b^{2}}{16 c^{3}}+\frac{\left(b^{2}-9\right)\left(\frac{\left(b^{2}-9\right)^{2} b^{2}}{4 c^{2}}+\frac{\left(b^{2}-9\right)^{2}\left(\frac{3 b^{2}}{2}-\frac{9}{2}\right)}{8 c^{2}}\right)}{4 c}+\frac{\left(b^{2}-9\right)^{4}}{256 c^{3}}\right) x^{3}}{3}+\frac{5\left(b^{2}-9\right)^{4} b x^{2}}{512 c^{4}}+\frac{\left(b^{2}-9\right)^{5} x}{1024 c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-9)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-9)*c^2*b+b*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2
)+c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c))*x^8+1/7*(1/4*(b^2-9)/c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2)+b*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+c*(1/8*(b^2-9)^2+2*(
b^2-9)*b^2+(3/2*b^2-9/2)^2))*x^7+1/6*(1/4*(b^2-9)/c*((b^2-9)*c*b+4*(3/2*b^2
-9/2)*b*c)+b*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+c*(1/4*(b^2-9)^2
/c*b+(b^2-9)/c*b*(3/2*b^2-9/2)))*x^6+1/5*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2+2*(b
^2-9)*b^2+(3/2*b^2-9/2)^2)+b*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+
c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-9)/c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+b*(1/8*(b^2-9)^2/c^2*(
3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16/c^2*(b^2-9)^3*b)*x^4+1/3*(1/4*(b^2
```

```
-9)/c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16*b^2*(b^2
-9)^3/c^3+1/256/c^3*(b^2-9)^4)*x^3+5/512*(b^2-9)^4/c^4*b*x^2+1/1024*(b^2-9)
^5/c^5*x
```

maxima [B] time = 1.41, size = 234, normalized size = 2.15

$$\frac{1}{11}\,c^{5}x^{11}+\frac{1}{2}\,bc^{4}x^{10}+\frac{10}{9}\,b^{2}c^{3}x^{9}+\frac{5}{4}\,b^{3}c^{2}x^{8}+\frac{5}{7}\,b^{4}cx^{7}+\frac{1}{6}\,b^{5}x^{6}+\frac{5\left(2\,cx^{3}+3\,bx^{2}\right)\left(b^{2}-9\right)^{4}}{1536\,c^{4}}+\frac{\left(6\,c^{2}x^{5}+15\,bcx^{4}+10\,b^{2}x^{3}\right)\left(b^{2}-9\right)^{3}}{192\,c^{3}}+\frac{\left(20\,c^{3}x^{7}+70\,bc^{2}x^{6}+84\,b^{2}cx^{5}+35\,b^{3}x^{4}\right)\left(b^{2}-9\right)^{2}}{224\,c^{2}}+\frac{\left(70\,c^{4}x^{9}+315\,bc^{3}x^{8}+540\,b^{2}c^{2}x^{7}+420\,b^{3}cx^{6}+126\,b^{4}x^{5}\right)\left(b^{2}-9\right)}{504\,c}+\frac{\left(b^{2}-9\right)^{5}x}{1024\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 9)/c + 1/1024*(b^2 - 9)^5*x/c^5
```

mupad [B] time = 0.34, size = 176, normalized size = 1.61

$$\frac{c^{5}\,x^{11}}{11}+\frac{5\,c^{3}\,x^{9}\,\left(b^{2}-1\right)}{4}+\frac{x\left(b^{2}-9\right)^{5}}{1024\,c^{5}}+\frac{3\,b\,x^{6}\,\left(7\,b^{4}-70\,b^{2}+135\right)}{16}+\frac{15\,c\,x^{7}\,\left(7\,b^{4}-42\,b^{2}+27\right)}{56}+\frac{b\,c^{4}\,x^{10}}{2}+\frac{3\,x^{5}\,\left(7\,b^{6}-105\,b^{4}+405\,b^{2}-243\right)}{32\,c}+\frac{15\,b\,c^{2}\,x^{8}\,\left(b^{2}-3\right)}{8}+\frac{15\,x^{3}\,\left(b^{2}-1\right)\,\left(b^{2}-9\right)^{3}}{256\,c^{3}}+\frac{5\,b\,x^{2}\left(b^{2}-9\right)^{4}}{512\,c^{4}}+\frac{15\,b\,x^{4}\,\left(b^{2}-3\right)\,\left(b^{2}-9\right)^{2}}{64\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 9/4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (5*c^3*x^9*(b^2 - 1))/4 + (x*(b^2 - 9)^5)/(1024*c^5) + (3*b
*x^6*(7*b^4 - 70*b^2 + 135))/16 + (15*c*x^7*(7*b^4 - 42*b^2 + 27))/56 + (b*
c^4*x^10)/2 + (3*x^5*(405*b^2 - 105*b^4 + 7*b^6 - 243))/(32*c) + (15*b*c^2*
x^8*(b^2 - 3))/8 + (15*x^3*(b^2 - 1)*(b^2 - 9)^3)/(256*c^3) + (5*b*x^2*(b^2
 - 9)^4)/(512*c^4) + (15*b*x^4*(b^2 - 3)*(b^2 - 9)^2)/(64*c^2)
```

sympy [B] time = 0.20, size = 253, normalized size = 2.32

$$\frac{bc^{4}x^{10}}{2}+\frac{c^{5}x^{11}}{11}+x^{9}\left(\frac{5b^{2}c^{3}}{4}-\frac{5c^{3}}{4}\right)+x^{8}\left(\frac{15b^{3}c^{2}}{8}-\frac{45bc^{2}}{8}\right)+x^{7}\left(\frac{15b^{4}c}{8}-\frac{45b^{2}c}{4}+\frac{405c}{56}\right)+x^{6}\left(\frac{21b^{5}}{16}-\frac{105b^{3}}{8}+\frac{405b}{16}\right)+\frac{x^{5}\left(21b^{6}-315b^{4}+1215b^{2}-729\right)}{32c}+\frac{x^{4}\left(15b^{7}-315b^{5}+2025b^{3}-3645b\right)}{64c^{2}}+\frac{x^{3}\left(15b^{8}-420b^{6}+4050b^{4}-14580b^{2}+10935\right)}{256c^{3}}+\frac{x^{2}\left(5b^{9}-180b^{7}+2430b^{5}-14580b^{3}+32805b\right)}{512c^{4}}+\frac{x\left(b^{10}-45b^{8}+810b^{6}-7290b^{4}+32805b^{2}-59049\right)}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(15
*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56) +
 x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 + 121
5*b**2 - 729)/(32*c) + x**4*(15*b**7 - 315*b**5 + 2025*b**3 - 3645*b)/(64*c
**2) + x**3*(15*b**8 - 420*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256*c**3
) + x**2*(5*b**9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*c**4)
+ x*(b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(1024*c**
5)
```

3.58 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4}{3c^6}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {610, 43}

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) + (5*(4 - b
- 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6) + (4 - b - 2*c*x)^10
/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 610

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-4+b)+cx\right)^5 \left(\frac{4+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(1024\left(\frac{1}{2}(-4+b)+cx\right)^5+1280\left(\frac{1}{2}(-4+b)+cx\right)^6+640\left(\frac{1}{2}(-4+b)+cx\right)^7+\right.}{c^5}$$
$$= \frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}+\frac{(4-b}{5}$$

Mathematica [A] time = 0.04, size = 207, normalized size = 1.90

$$\frac{5}{8}\left(3b^3-16b\right)c^2x^8+\frac{\left(b^2-16\right)^5x}{1024c^5}+\frac{5b\left(b^2-16\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-16\right)c^3x^9+\frac{5\left(b^2-16\right)^3\left(9b^2-16\right)x^3}{768c^3}+\frac{5b\left(b^2-16\right)^2\left(3b^2-16\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-224b^2+256\right)cx^7+\frac{\left(b^2-16\right)\left(21b^4-224b^2+256\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-1120b^2+3840\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16
+ b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*x^
4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(3840
- 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 + (5*(
-16*b + 3*b^3)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 +
 (c^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[((-16 + b^2)/(4*c) + b*x + c*x^2)^5, x]
```

fricas [B] time = 0.39, size = 235, normalized size = 2.16

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\left(9\,b^2-16\right)c^8x^9+443520\left(3\,b^3-16\,b\right)c^7x^8+63360\left(21\,b^4-224\,b^2+256\right)c^6x^7+14784\left(63\,b^5-1120\,b^3+3840\,b\right)c^5x^6+22176\left(21\,b^6-560\,b^4+3840\,b^2-4096\right)c^4x^5+55440\left(3\,b^7-112\,b^5+1280\,b^3-4096\,b\right)c^3x^4+4620\left(9\,b^8-448\,b^6+7680\,b^4-49152\,b^2+65536\right)c^2x^3+6930\left(b^9-64\,b^7+1536\,b^5-16384\,b^3+65536\,b\right)cx^2+693\left(b^{10}-80\,b^8+2560\,b^6-40960\,b^4+327680\,b^2-1048576\right)x}{709632\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9
+ 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7 +
14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3840
*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*x^4
+ 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930*(b^9
 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^8 + 25
60*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5

giac [B] time = 0.53, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 1576960\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 7096320\,bc^{7}x^{8} + 465696\,b^{6}c^{4}x^{5} - 14192640\,b^{2}c^{6}x^{7} + 166320\,b^{7}c^{3}x^{4} - 16558080\,b^{3}c^{5}x^{6} + 41580\,b^{8}c^{2}x^{3} - 12418560\,b^{4}c^{4}x^{5} + 16220160\,c^{6}x^{7} + 6930\,b^{9}cx^{2} - 6209280\,b^{5}c^{3}x^{4} + 56770560\,bc^{5}x^{6} + 693\,b^{10}x - 2069760\,b^{6}c^{2}x^{3} + 85155840\,b^{2}c^{4}x^{5} - 443520\,b^{7}cx^{2} + 70963200\,b^{3}c^{3}x^{4} - 55440\,b^{8}x + 35481600\,b^{4}c^{2}x^{3} - 90832896\,c^{4}x^{5} + 10644480\,b^{5}cx^{2} - 227082240\,bc^{3}x^{4} + 1774080\,b^{6}x - 227082240\,b^{2}c^{2}x^{3} - 113541120\,b^{3}cx^{2} - 28385280\,b^{4}x + 302776320\,c^{2}x^{3} + 454164480\,bcx^{2} + 227082240\,b^{2}x - 726663168\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^6
- 7096320*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 14192640*b^2*c^6*x^7 + 166320*b^
7*c^3*x^4 - 16558080*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 12418560*b^4*c^4*x^5
 + 16220160*c^6*x^7 + 6930*b^9*c*x^2 - 6209280*b^5*c^3*x^4 + 56770560*b*c^5
*x^6 + 693*b^10*x - 2069760*b^6*c^2*x^3 + 85155840*b^2*c^4*x^5 - 443520*b^7
*c*x^2 + 70963200*b^3*c^3*x^4 - 55440*b^8*x + 35481600*b^4*c^2*x^3 - 908328
96*c^4*x^5 + 10644480*b^5*c*x^2 - 227082240*b*c^3*x^4 + 1774080*b^6*x - 227
082240*b^2*c^2*x^3 - 113541120*b^3*c*x^2 - 28385280*b^4*x + 302776320*c^2*x
^3 + 454164480*b*c*x^2 + 227082240*b^2*x - 726663168*x)/c^5

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^{5}x^{11}}{11}+\frac{b\,c^{4}x^{10}}{2}+\frac{\left(4b^{2}c^{3}+\frac{\left(b^{2}-16\right)c^{3}}{4}+\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-8\right)c^{2}\right)c\right)x^{9}}{9}+\frac{\left(\left(b^{2}-16\right)b\,c^{2}+\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-8\right)c^{2}\right)b+\left(\left(b^{2}-16\right)bc+4\left(\frac{3b^{2}}{2}-8\right)bc\right)c\right)x^{8}}{8}+\frac{\left(\left(\left(b^{2}-16\right)bc+4\left(\frac{3b^{2}}{2}-8\right)bc\right)b+\left(2\left(b^{2}-16\right)b^{2}+\frac{\left(b^{2}-16\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-8\right)^{2}\right)c+\frac{\left(b^{2}-16\right)\left(4b^{2}c^{2}+2\left(\frac{3b^{2}}{2}-8\right)c^{2}\right)}{4c}\right)x^{7}}{7}+\frac{\left(\left(2\left(b^{2}-16\right)b^{2}+\frac{\left(b^{2}-16\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-8\right)^{2}\right)b+\left(\frac{\left(b^{2}-16\right)^{2}b}{4c}+\frac{\left(b^{2}-16\right)\left(\frac{3b^{2}}{2}-8\right)b}{c}\right)c+\frac{\left(b^{2}-16\right)\left(\left(b^{2}-16\right)bc+4\left(\frac{3b^{2}}{2}-8\right)bc\right)}{4c}\right)x^{6}}{6}+\frac{\left(\left(\frac{\left(b^{2}-16\right)^{2}b}{4c}+\frac{\left(b^{2}-16\right)\left(\frac{3b^{2}}{2}-8\right)b}{c}\right)b+\left(\frac{\left(b^{2}-16\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-16\right)^{2}\left(\frac{3b^{2}}{2}-8\right)}{8c^{2}}\right)c+\frac{\left(b^{2}-16\right)\left(2\left(b^{2}-16\right)b^{2}+\frac{\left(b^{2}-16\right)^{2}}{8}+\left(\frac{3b^{2}}{2}-8\right)^{2}\right)}{4c}\right)x^{5}}{5}+\frac{\left(\left(\frac{\left(b^{2}-16\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-16\right)^{2}\left(\frac{3b^{2}}{2}-8\right)}{8c^{2}}\right)b+\frac{\left(b^{2}-16\right)^{3}b}{16c^{2}}+\frac{\left(b^{2}-16\right)\left(\frac{\left(b^{2}-16\right)^{2}b}{4c}+\frac{\left(b^{2}-16\right)\left(\frac{3b^{2}}{2}-8\right)b}{c}\right)}{4c}\right)x^{4}}{4}+\frac{\left(\frac{\left(b^{2}-16\right)^{3}b^{2}}{16c^{3}}+\frac{\left(b^{2}-16\right)\left(\frac{\left(b^{2}-16\right)^{2}b^{2}}{4c^{2}}+\frac{\left(b^{2}-16\right)^{2}\left(\frac{3b^{2}}{2}-8\right)}{8c^{2}}\right)}{4c}+\frac{\left(b^{2}-16\right)^{4}}{256c^{3}}\right)x^{3}}{3}+\frac{5\left(b^{2}-16\right)^{4}b\,x^{2}}{512c^{4}}+\frac{\left(b^{2}-16\right)^{5}x}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x)

[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-16)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-
8)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-16)*c^2*b+b*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+
c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c))*x^8+1/7*(1/4*(b^2-16)/c*(2*(3/2*b^2-8)*
c^2+4*b^2*c^2)+b*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2))*x^7+1/6*(1/4*(b^2-16)/c*((b^2-16)*c*b+4*(3/2*b^2-8)
*b*c)+b*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+c*(1/4*(b^2-16)^2/c*b
+(b^2-16)/c*b*(3/2*b^2-8)))*x^6+1/5*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2)+b*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+c*(1
/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-16
)/c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+b*(1/8*(b^2-16)^2/c^2*(3/
2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16/c^2*(b^2-16)^3*b)*x^4+1/3*(1/4*(b^2-1

```
6)/c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16*b^2*(b^2-
16)^3/c^3+1/256/c^3*(b^2-16)^4)*x^3+5/512*(b^2-16)^4/c^4*b*x^2+1/1024*(b^2-
16)^5/c^5*x
```

maxima [B] time = 1.40, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6+\frac{5\left(2cx^3+3bx^2\right)\left(b^2-16\right)^4}{1536c^4}+\frac{\left(6c^2x^5+15bcx^4+10b^2x^3\right)\left(b^2-16\right)^3}{192c^3}+\frac{\left(20c^3x^7+70bc^2x^6+84b^2cx^5+35b^3x^4\right)\left(b^2-16\right)^2}{224c^2}+\frac{\left(70c^4x^9+315bc^3x^8+540b^2c^2x^7+420b^3cx^6+126b^4x^5\right)\left(b^2-16\right)}{504c}+\frac{\left(b^2-16\right)^5x}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 16)^4/c^4 + 1/19
2*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 16)^3/c^3 + 1/224*(20*c^3*x^
7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*(70*
c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b
^2 - 16)/c + 1/1024*(b^2 - 16)^5*x/c^5
```

mupad [B] time = 0.32, size = 184, normalized size = 1.69

$$\frac{c^5x^{11}}{11}+\frac{x\left(b^2-16\right)^5}{1024c^5}+\frac{bx^6\left(63b^4-1120b^2+3840\right)}{48}+\frac{5cx^7\left(21b^4-224b^2+256\right)}{56}+\frac{bc^4x^{10}}{2}+\frac{5c^3x^9\left(9b^2-16\right)}{36}+\frac{x^5\left(21b^6-560b^4+3840b^2-4096\right)}{32c}+\frac{5bc^2x^8\left(3b^2-16\right)}{8}+\frac{5bx^2\left(b^2-16\right)^4}{512c^4}+\frac{5x^3\left(b^2-16\right)^3\left(9b^2-16\right)}{768c^3}+\frac{5bx^4\left(b^2-16\right)^2\left(3b^2-16\right)}{64c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 16)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 1120*b^2 + 3
840))/48 + (5*c*x^7*(21*b^4 - 224*b^2 + 256))/56 + (b*c^4*x^10)/2 + (5*c^3*
x^9*(9*b^2 - 16))/36 + (x^5*(3840*b^2 - 560*b^4 + 21*b^6 - 4096))/(32*c) +
(5*b*c^2*x^8*(3*b^2 - 16))/8 + (5*b*x^2*(b^2 - 16)^4)/(512*c^4) + (5*x^3*(b
^2 - 16)^3*(9*b^2 - 16))/(768*c^3) + (5*b*x^4*(b^2 - 16)^2*(3*b^2 - 16))/(6
4*c^2)
```

sympy [B] time = 0.20, size = 248, normalized size = 2.28

$$\frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\left(\frac{5b^2c^3}{4}-\frac{20c^3}{9}\right)+x^8\left(\frac{15b^3c^2}{8}-10bc^2\right)+x^7\left(\frac{15b^4c}{8}-20b^2c+\frac{160c}{7}\right)+x^6\left(\frac{21b^5}{16}-\frac{70b^3}{3}+80b\right)+\frac{x^5\left(21b^6-560b^4+3840b^2-4096\right)}{32c}+\frac{x^4\left(15b^7-560b^5+6400b^3-20480b\right)}{64c^2}+\frac{x^3\left(45b^8-2240b^6+38400b^4-245760b^2+327680\right)}{768c^3}+\frac{x^2\left(5b^9-320b^7+7680b^5-81920b^3+327680b\right)}{512c^4}+\frac{x\left(b^{10}-80b^8+2560b^6-40960b^4+327680b^2-1048576\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9) + x**8*(1
5*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**2*c + 160*c/7) + x**
6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*b**6 - 560*b**4 + 3840*b**2 -
4096)/(32*c) + x**4*(15*b**7 - 560*b**5 + 6400*b**3 - 20480*b)/(64*c**2) +
x**3*(45*b**8 - 2240*b**6 + 38400*b**4 - 245760*b**2 + 327680)/(768*c**3) +
 x**2*(5*b**9 - 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c**4) +
x*(b**10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)/(1024*
c**5)
```

3.59 $\int \frac{1}{2+4x+3x^2} \, dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-1),x]
```

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2+4x+3x^2} \, dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-8-x^2} \, dx, x, 4+6x\right)\right) \\
&= \frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-1),x]
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+4x+3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 4*x + 3*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(2 + 4*x + 3*x^2)^(-1), x]
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

giac [A] time = 0.37, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

maple [A] time = 0.04, size = 17, normalized size = 0.94

$$\frac{\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

maxima [A] time = 2.95, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

mupad [B] time = 0.14, size = 16, normalized size = 0.89

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(3x+2)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*x + 2))/2))/2
```

sympy [A] time = 0.11, size = 22, normalized size = 1.22

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2),x)
```

```
[Out] sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

3.60 $\int \frac{1}{4-2\sqrt{3}x+x^2} dx$

Optimal. Leaf size=12

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {618, 204}

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-2\sqrt{3}x+x^2} dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2} dx, x, -2\sqrt{3}+2x\right)\right)\\ &= -\tan^{-1}\left(\sqrt{3}-x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
[Out] -ArcTan[Sqrt[3] - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-2\sqrt{3}\,x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
[Out] IntegrateAlgebraic[(4 - 2*Sqrt[3]*x + x^2)^(-1), x]
```

fricas [A] time = 0.40, size = 8, normalized size = 0.67

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="fricas")
[Out] arctan(x - sqrt(3))
```

giac [A] time = 0.41, size = 8, normalized size = 0.67

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="giac")
[Out] arctan(x - sqrt(3))
```

maple [A] time = 0.10, size = 9, normalized size = 0.75

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x^2-2*3^(1/2)*x),x)
[Out] arctan(x-3^(1/2))
```

maxima [A] time = 2.97, size = 8, normalized size = 0.67

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(x - sqrt(3))
```

mupad [B] time = 0.27, size = 8, normalized size = 0.67

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*3^(1/2)*x + 4),x)
```

```
[Out] atan(x - 3^(1/2))
```

sympy [A] time = 0.16, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

```
[Out] atan(x - sqrt(3))
```

3.61 $\int \frac{1}{2+4x-3x^2} \, dx$

Optimal. Leaf size=19

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{2+4x-3x^2} \, dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{40-x^2} \, dx, x, 4-6x\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.79

$$\frac{\log\left(3x+\sqrt{10}-2\right)-\log\left(-3x+\sqrt{10}+2\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-1),x]
[Out] (-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+4x-3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 4*x - 3*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(2 + 4*x - 3*x^2)^(-1), x]
```

fricas [B] time = 0.38, size = 39, normalized size = 2.05

$$\frac{1}{20}\,\sqrt{10}\,\log\left(\frac{9\,x^2+2\,\sqrt{10}\,(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="fricas")
[Out] 1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x -
 2))
```

giac [A] time = 0.46, size = 31, normalized size = 1.63

$$-\frac{1}{20}\,\sqrt{10}\,\log\left(\frac{\left|6\,x-2\,\sqrt{10}-4\right|}{\left|6\,x+2\,\sqrt{10}-4\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="giac")
[Out] -1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))
```

maple [A] time = 0.04, size = 17, normalized size = 0.89

$$\frac{\sqrt{10}\ \operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2),x)
```

```
[Out] 1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

maxima [A] time = 2.95, size = 27, normalized size = 1.42

$$-\frac{1}{20}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

mupad [B] time = 0.21, size = 15, normalized size = 0.79

$$\frac{\sqrt{10}\ \operatorname{atanh}\left(\sqrt{10}\ \left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2),x)
```

```
[Out] (10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/10
```

sympy [A] time = 0.12, size = 39, normalized size = 2.05

$$\frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20}-\frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2),x)
```

```
[Out] sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)/
20
```

3.62 $\int \frac{1}{2+5x+3x^2} dx$

Optimal. Leaf size=13

$$\log(3x+2) - \log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\log(3x+2) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x+3x^2} dx &= 3\int \frac{1}{2+3x} dx - 3\int \frac{1}{3+3x} dx \\ &= -\log(1+x) + \log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(3x+2) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+5x+3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 5*x + 3*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(2 + 5*x + 3*x^2)^(-1), x]
```

fricas [A] time = 0.39, size = 13, normalized size = 1.00

$$\log(3x+2) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")
[Out] log(3*x + 2) - log(x + 1)
```

giac [A] time = 0.51, size = 15, normalized size = 1.15

$$\log(|3x+2|) - \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="giac")
[Out] log(abs(3*x + 2)) - log(abs(x + 1))
```

maple [A] time = 0.05, size = 14, normalized size = 1.08

$$\ln(3x+2) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2),x)
[Out] -ln(x+1)+ln(3*x+2)
```

maxima [A] time = 1.33, size = 13, normalized size = 1.00

$$\log(3x+2) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")
```

```
[Out] log(3*x + 2) - log(x + 1)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.62

$$-2\,\mathrm{atanh}\left(6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2),x)
```

```
[Out] -2*atanh(6*x + 5)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log\left(x+\frac{2}{3}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2),x)
```

```
[Out] log(x + 2/3) - log(x + 1)
```

3.63 $\int \frac{1}{2+5x-3x^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x-3x^2}\,dx &= -\left(\frac{3}{7}\int \frac{1}{-1-3x}\,dx\right)+\frac{3}{7}\int \frac{1}{6-3x}\,dx\\ &= -\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] -1/7*Log[2 - x] + Log[1 + 3*x]/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+5x-3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 + 5*x - 3*x^2)^(-1), x]
```

fricas [A] time = 0.40, size = 15, normalized size = 0.71

$$\frac{1}{7}\,\log\left(3\,x+1\right)-\frac{1}{7}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="fricas")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

giac [A] time = 0.46, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(|3\,x+1|\right)-\frac{1}{7}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="giac")
```

```
[Out] 1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))
```

maple [A] time = 0.05, size = 16, normalized size = 0.76

$$\frac{\ln\left(3x+1\right)}{7}-\frac{\ln\left(x-2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2),x)
```

```
[Out] -1/7*ln(x-2)+1/7*ln(1+3*x)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.71

$$\frac{1}{7}\,\log\left(3\,x+1\right)-\frac{1}{7}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="maxima")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

mupad [B] time = 0.09, size = 8, normalized size = 0.38

$$\frac{2\,\mathrm{atanh}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2),x)
```

```
[Out] (2*atanh((6*x)/7 - 5/7))/7
```

sympy [A] time = 0.11, size = 14, normalized size = 0.67

$$-\frac{\log\left(x-2\right)}{7}+\frac{\log\left(x+\frac{1}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2),x)
```

```
[Out] -log(x - 2)/7 + log(x + 1/3)/7
```

3.64 $\int \frac{1}{3+4x+x^2} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(x+2)$$

Rubi [B] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {616, 31}

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + x^2)^(-1),x]

[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{3+4x+x^2} \, dx &= \frac{1}{2}\int \frac{1}{1+x} \, dx - \frac{1}{2}\int \frac{1}{3+x} \, dx \\ &= \frac{1}{2}\log(1+x) - \frac{1}{2}\log(3+x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 17, normalized size = 2.83

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + x^2)^(-1),x]
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3+4x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*x + x^2)^(-1),x]
[Out] IntegrateAlgebraic[(3 + 4*x + x^2)^(-1), x]
```

fricas [B] time = 0.40, size = 13, normalized size = 2.17

$$-\frac{1}{2}\,\log\left(x+3\right)+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="fricas")
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

giac [B] time = 0.33, size = 15, normalized size = 2.50

$$-\frac{1}{2}\,\log\left(|x+3|\right)+\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="giac")
[Out] -1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))
```

maple [B] time = 0.05, size = 14, normalized size = 2.33

$$-\frac{\ln\left(x+3\right)}{2}+\frac{\ln\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x+3),x)
[Out] 1/2*ln(x+1)-1/2*ln(3+x)
```

maxima [B] time = 1.29, size = 13, normalized size = 2.17

$$-\frac{1}{2}\,\log\left(x+3\right)+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

mupad [B] time = 0.18, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^2 + 3),x)
```

```
[Out] -atanh(x + 2)
```

sympy [B] time = 0.10, size = 12, normalized size = 2.00

$$\frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x+3),x)
```

```
[Out] log(x + 1)/2 - log(x + 3)/2
```

3.65 $\int \frac{1}{1+\pi x+2x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+2x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-8+\pi^2-x^2}\,dx, x, \pi+4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\pi x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + Pi*x + 2*x^2)^(-1), x]
```

fricas [B] time = 0.42, size = 50, normalized size = 1.85

$$\frac{\log\left(\frac{\pi^2+4\,\pi x+8\,x^2-(\pi+4\,x)\sqrt{\pi^2-8}-4}{\pi x+2\,x^2+1}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")
```

```
[Out] log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2 +
 1))/sqrt(pi^2 - 8)
```

giac [A] time = 0.35, size = 40, normalized size = 1.48

$$\frac{\log\left(\frac{\left|\pi+4\,x-\sqrt{\pi^2-8}\right|}{\left|\pi+4\,x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")
```

```
[Out] log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^
2 - 8)
```

maple [A] time = 0.06, size = 24, normalized size = 0.89

$$-\frac{2\,\mathrm{arctanh}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+2*x^2+1),x)
```

```
[Out] -2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

maxima [A] time = 1.31, size = 38, normalized size = 1.41

$$\frac{\log\left(\frac{\pi+4\,x-\sqrt{\pi^2-8}}{\pi+4\,x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")
```

```
[Out] log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8)
```

mupad [B] time = 0.36, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi+4\,x}{\sqrt{\Pi^2-8}}\right)}{\sqrt{\Pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x + 2*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi + 4*x)/(Pi^2 - 8)^(1/2)))/(Pi^2 - 8)^(1/2)
```

sympy [B] time = 0.22, size = 76, normalized size = 2.81

$$\frac{\log\left(x-\frac{\pi^2}{4\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}-\frac{\log\left(x-\frac{2}{\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x**2+1),x)
```

```
[Out] log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/sqrt(-8 + p
i**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sqrt(-8 + pi**2)))/sqr
t(-8 + pi**2)
```

3.66 $\int \frac{1}{1+\pi x-2x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x-2x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8+\pi^2-x^2}\,dx, x, \pi-4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{4x-\pi}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
[Out] (2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\pi x-2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Pi*x - 2*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(1 + Pi*x - 2*x^2)^(-1), x]
```

fricas [B] time = 0.42, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-4\,\pi x+8\,x^2-(\pi-4\,x)\sqrt{\pi^2+8}+4}{\pi x-2\,x^2+1}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="fricas")
[Out] log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2
+ 1))/sqrt(pi^2 + 8)
```

giac [A] time = 0.38, size = 45, normalized size = 1.67

$$-\frac{\log\left(\frac{\left|-\pi+4\,x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4\,x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="giac")
[Out] -log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt(
pi^2 + 8)
```

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$\frac{2\,\mathrm{arctanh}\left(\frac{4x-\pi}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-2*x^2+1),x)
```

```
[Out] 2/(Pi^2+8)^(1/2)*arctanh((4*x-Pi)/(Pi^2+8)^(1/2))
```

maxima [A] time = 1.33, size = 39, normalized size = 1.44

$$-\frac{\log\left(\frac{\pi-4\,x+\sqrt{\pi^2+8}}{\pi-4\,x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqrt(pi^2 + 8
)
```

mupad [B] time = 0.39, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-4\,x}{\sqrt{\Pi^2+8}}\right)}{\sqrt{\Pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x - 2*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi - 4*x)/(Pi^2 + 8)^(1/2)))/(Pi^2 + 8)^(1/2)
```

sympy [B] time = 0.23, size = 76, normalized size = 2.81

$$-\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x**2+1),x)
```

```
[Out] -log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sqrt(8 + pi*
*2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(8 + pi**2)))/sqrt(8
+ pi**2)
```

3.67 $\int \frac{1}{1+\pi x+3x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+3x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-12+\pi^2-x^2}\,dx, x, \pi+6x\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\pi x+3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + Pi*x + 3*x^2)^(-1), x]
```

fricas [A] time = 0.41, size = 41, normalized size = 1.32

$$\frac{2\sqrt{-\pi^2+12}\,\arctan\left(\frac{(\pi+6\,x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2-12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 -
12)
```

giac [A] time = 0.48, size = 27, normalized size = 0.87

$$\frac{2\,\arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

maple [A] time = 0.05, size = 28, normalized size = 0.90

$$\frac{2\arctan\left(\frac{6x+\pi}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+3*x^2+1),x)
```

```
[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

maxima [A] time = 1.32, size = 27, normalized size = 0.87

$$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

mupad [B] time = 0.38, size = 23, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi+6x}{\sqrt{\Pi^2-12}}\right)}{\sqrt{\Pi^2-12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x + 3*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi + 6*x)/(Pi^2 - 12)^(1/2)))/(Pi^2 - 12)^(1/2)
```

sympy [C] time = 0.20, size = 87, normalized size = 2.81

$$-\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{12-\pi^2}}+\frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{12-\pi^2}}+\frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x**2+1),x)
```

```
[Out] -I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqrt
(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(12
- pi**2))/sqrt(12 - pi**2)
```

3.68 $\int \frac{1}{1+\pi x-3x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1+\pi x-3x^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{12+\pi^2-x^2}\,dx, x, \pi-6x\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{6x-\pi}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\pi x-3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + Pi*x - 3*x^2)^(-1), x]
```

fricas [B] time = 0.40, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-6\,\pi x+18\,x^2-(\pi-6\,x)\sqrt{\pi^2+12}+6}{\pi x-3\,x^2+1}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="fricas")
```

```
[Out] log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x^
2 + 1))/sqrt(pi^2 + 12)
```

giac [A] time = 0.53, size = 45, normalized size = 1.67

$$-\frac{\log\left(\frac{\left|-\pi+6\,x-\sqrt{\pi^2+12}\right|}{\left|-\pi+6\,x+\sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sqr
t(pi^2 + 12)
```

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$\frac{2\,\mathrm{arctanh}\left(\frac{6x-\pi}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-3*x^2+1),x)
```

```
[Out] 2/(Pi^2+12)^(1/2)*arctanh((6*x-Pi)/(Pi^2+12)^(1/2))
```

maxima [A] time = 1.44, size = 39, normalized size = 1.44

$$-\frac{\log\left(\frac{\pi-6\,x+\sqrt{\pi^2+12}}{\pi-6\,x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/sqrt(pi^2 +
 12)
```

mupad [B] time = 0.42, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-6\,x}{\sqrt{\Pi^2+12}}\right)}{\sqrt{\Pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x - 3*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi - 6*x)/(Pi^2 + 12)^(1/2)))/(Pi^2 + 12)^(1/2)
```

sympy [B] time = 0.23, size = 76, normalized size = 2.81

$$\frac{\log\left(x-\frac{\pi}{6}+\frac{\pi^2}{6\sqrt{\pi^2+12}}+\frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}-\frac{\log\left(x-\frac{\pi}{6}-\frac{2}{\sqrt{\pi^2+12}}-\frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x**2+1),x)
```

```
[Out] log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/sqrt(pi**2
+ 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sqrt(pi**2 + 12)))/sqr
t(pi**2 + 12)
```

3.69 $\int \frac{1}{a+cx+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+cx+bx^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-4ab+c^2-x^2}\,dx, x, c+2bx\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+cx+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x + b*x^2)^(-1), x]
```

fricas [A] time = 0.40, size = 113, normalized size = 2.97

$$\left[-\frac{\sqrt{-4\,ab+c^2}\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2-\sqrt{-4\,ab+c^2}\,(2\,bx+c)}{bx^2+cx+a}\right)}{4\,ab-c^2},\,-\frac{2\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="fricas")
```

```
[Out] [-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b +
 c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x + c)
/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]
```

giac [A] time = 0.43, size = 34, normalized size = 0.89

$$\frac{2\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

maple [A] time = 0.06, size = 35, normalized size = 0.92

$$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a),x)
```

```
[Out] 2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for mo
re details)Is c^2-4*a*b positive or negative?
```

mupad [B] time = 0.23, size = 46, normalized size = 1.21

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{4\,a\,b-c^2}}+\frac{2\,b\,x}{\sqrt{4\,a\,b-c^2}}\right)}{\sqrt{4\,a\,b-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x + b*x^2),x)
```

```
[Out] (2*atan(c/(4*a*b - c^2)^(1/2) + (2*b*x)/(4*a*b - c^2)^(1/2)))/(4*a*b - c^2)
^(1/2)
```

sympy [B] time = 0.22, size = 124, normalized size = 3.26

$$-\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)+\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a),x)
```

```
[Out] -sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sqr
t(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b*s
qrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))
```

3.70 $\int \frac{1}{b+2ax+bx^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b+2ax+bx^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right)-x^2}\,dx, x, 2a+2bx\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-1),x]
[Out] ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b+2ax+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*a*x + b*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(b + 2*a*x + b*x^2)^(-1), x]
```

fricas [A] time = 0.41, size = 124, normalized size = 3.54

$$\left[\frac{\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}\,(bx+a)}{bx^2+2\,ax+b}\right)}{2\,\sqrt{a^2-b^2}}, -\frac{\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(bx+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")
[Out] [1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b
*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]
```

giac [A] time = 0.37, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

maple [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b),x)
```

```
[Out] 1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.27, size = 33, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{a+b\,x}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x + b*x^2),x)
```

```
[Out] -atanh((a + b*x)/((a + b)^(1/2)*(a - b)^(1/2)))/((a + b)^(1/2)*(a - b)^(1/2
))
```

sympy [B] time = 0.23, size = 100, normalized size = 2.86

$$\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}-\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b),x)
```

```
[Out] sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b*
*2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a**
2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2
```

3.71 $\int \frac{1}{b+2ax-bx^2}\,dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b+2ax-bx^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2a-2bx\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.28

$$-\frac{\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b+2ax-bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*a*x - b*x^2)^(-1), x]
```

fricas [B] time = 0.39, size = 67, normalized size = 2.09

$$\frac{\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}\,(bx-a)}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="fricas")
```

```
[Out] 1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*
x^2 - 2*a*x - b))/sqrt(a^2 + b^2)
```

giac [A] time = 0.45, size = 55, normalized size = 1.72

$$-\frac{\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2
+ b^2)))/sqrt(a^2 + b^2)
```

maple [A] time = 0.07, size = 31, normalized size = 0.97

$$\frac{\operatorname{arctanh}\left(\frac{2bx-2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b),x)
```

```
[Out] 1/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 2.91, size = 49, normalized size = 1.53

$$-\frac{\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/sqrt(a^2
+ b^2)
```

mupad [B] time = 0.23, size = 28, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{a-b\,x}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x - b*x^2),x)
```

```
[Out] -atanh((a - b*x)/(a^2 + b^2)^(1/2))/(a^2 + b^2)^(1/2)
```

sympy [B] time = 0.25, size = 102, normalized size = 3.19

$$-\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}+\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b),x)
```

```
[Out] -sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a - b**2*sqrt
(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x + (a**2*sqrt(1/(a**2
+ b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)/2
```

3.72 $\int \frac{1}{(2+4x+3x^2)^2} \, dx$

Optimal. Leaf size=43

$$\frac{3x+2}{4(3x^2+4x+2)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {614, 618, 204}

$$\frac{3x+2}{4(3x^2+4x+2)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{2+3x}{4(2+4x+3x^2)} + \frac{3}{4}\int \frac{1}{2+4x+3x^2}\,dx$$
$$= \frac{2+3x}{4(2+4x+3x^2)} - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 4+6x\right)$$
$$= \frac{2+3x}{4(2+4x+3x^2)} + \frac{3\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{3x+2}{4(3x^2+4x+2)} + \frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-2),x]
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 4*x + 3*x^2)^(-2),x]
[Out] IntegrateAlgebraic[(2 + 4*x + 3*x^2)^(-2), x]
```

fricas [A] time = 0.39, size = 45, normalized size = 1.05

$$\frac{3\sqrt{2}\,(3x^2+4x+2)\arctan\left(\frac{1}{2}\sqrt{2}\,(3x+2)\right) + 6x + 4}{8(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/(
3*x^2 + 4*x + 2)
```

giac [A] time = 0.58, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)+\frac{3\,x+2}{4\,\left(3\,x^2+4\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

maple [A] time = 0.04, size = 37, normalized size = 0.86

$$\frac{3\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{8}+\frac{6x+4}{24x^2+32x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^2,x)
```

```
[Out] 1/8*(6*x+4)/(3*x^2+4*x+2)+3/8*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

maxima [A] time = 2.27, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)+\frac{3\,x+2}{4\,\left(3\,x^2+4\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.77

$$\frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4\,x}{3}+\frac{2}{3}}+\frac{3\,\sqrt{2}\,\mathrm{atan}\left(\frac{3\,\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2)^2,x)
```

```
[Out] (x/4 + 1/6)/((4*x)/3 + x^2 + 2/3) + (3*2^(1/2)*atan((3*2^(1/2)*x)/2 + 2^(1/
2)))/8
```

sympy [A] time = 0.15, size = 39, normalized size = 0.91

$$\frac{3x+2}{12x^2+16x+8}+\frac{3\sqrt{2}\,\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**2,x)
```

```
[Out] (3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/8
```

3.73 $\int \frac{1}{\left(2+4x-3x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {614, 618, 206}

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqr
t[10])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(2+4x-3x^2\right)^2}\,dx = -\frac{2-3x}{20\left(2+4x-3x^2\right)} + \frac{3}{20}\int \frac{1}{2+4x-3x^2}\,dx$$
$$= -\frac{2-3x}{20\left(2+4x-3x^2\right)} - \frac{3}{10}\,\mathrm{Subst}\left(\int \frac{1}{40-x^2}\,dx, x, 4-6x\right)$$
$$= -\frac{2-3x}{20\left(2+4x-3x^2\right)} - \frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.44

$$\frac{2-3x}{20\left(3x^2-4x-2\right)} - \frac{3\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}} + \frac{3\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] (2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[10
]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2+4x-3x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(2 + 4*x - 3*x^2)^(-2), x]
```

fricas [A] time = 0.39, size = 68, normalized size = 1.58

$$\frac{3\sqrt{10}\left(3x^2-4x-2\right)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-60x+40}{400\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)
```

giac [A] time = 0.52, size = 51, normalized size = 1.19

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}-4|}{|6x+2\sqrt{10}-4|}\right)-\frac{3x-2}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] -3/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

maple [A] time = 0.06, size = 37, normalized size = 0.86

$$\frac{3\sqrt{10}\ \operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}-\frac{6x-4}{40\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^2,x)
```

```
[Out] -1/40*(6*x-4)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

maxima [A] time = 2.81, size = 47, normalized size = 1.09

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)-\frac{3x-2}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] -3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(3*x
- 2)/(3*x^2 - 4*x - 2)
```

mupad [B] time = 0.16, size = 34, normalized size = 0.79

$$\frac{3\sqrt{10}\ \operatorname{atanh}\left(\sqrt{10}\ \left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200}+\frac{\frac{x}{20}-\frac{1}{30}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2)^2,x)
```

```
[Out] (3*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + (x/20 - 1/30)/((4*x)/3
- x^2 + 2/3)
```

sympy [A] time = 0.15, size = 58, normalized size = 1.35

$$\frac{2-3x}{60x^2-80x-40}+\frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] (2 - 3*x)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400
- 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

3.74 $\int \frac{1}{(2+5x+3x^2)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {614, 616, 31}

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2)^(-2),x]
```

```
[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{5+6x}{2+5x+3x^2} - 6\int \frac{1}{2+5x+3x^2}\,dx$$
$$= -\frac{5+6x}{2+5x+3x^2} - 18\int \frac{1}{2+3x}\,dx + 18\int \frac{1}{3+3x}\,dx$$
$$= -\frac{5+6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.97

$$\frac{-6x-5}{3x^2+5x+2} + 6\log(x+1) - 6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-2),x]
[Out] (-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 5*x + 3*x^2)^(-2),x]
[Out] IntegrateAlgebraic[(2 + 5*x + 3*x^2)^(-2), x]
```

fricas [A] time = 0.38, size = 53, normalized size = 1.56

$$-\frac{6\,(3\,x^2+5\,x+2)\log(3\,x+2) - 6\,(3\,x^2+5\,x+2)\log(x+1) + 6\,x + 5}{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")
[Out] -(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x +
 5)/(3*x^2 + 5*x + 2)
```

giac [A] time = 0.45, size = 36, normalized size = 1.06

$$-\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(|3\,x+2|) + 6\,\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))
```

maple [A] time = 0.05, size = 32, normalized size = 0.94

$$-6\ln(3x+2)+6\ln(x+1)-\frac{3}{3x+2}-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^2,x)
[Out] -3/(3*x+2)-6*ln(3*x+2)-1/(x+1)+6*ln(x+1)
```

maxima [A] time = 1.32, size = 34, normalized size = 1.00

$$-\frac{6\,x+5}{3\,x^2+5\,x+2}-6\,\log(3\,x+2)+6\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

mupad [B] time = 0.20, size = 34, normalized size = 1.00

$$-6\,\ln\left(\frac{3\,x+2}{x+1}\right)-\frac{2\,\left(3\,x+\frac{5}{2}\right)}{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2)^2,x)
[Out] - 6*log((3*x + 2)/(x + 1)) - (2*(3*x + 5/2))/(5*x + 3*x^2 + 2)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.91

$$\frac{-6x-5}{3x^2+5x+2}-6\log\left(x+\frac{2}{3}\right)+6\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**2,x)
[Out] (-6*x - 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

3.75 $\int \frac{1}{\left(2+5x-3x^2\right)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {614, 616, 31}

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+5x-3x^2\right)^2}\,dx &= -\frac{5-6x}{49\left(2+5x-3x^2\right)}+\frac{6}{49}\int \frac{1}{2+5x-3x^2}\,dx\\ &= -\frac{5-6x}{49\left(2+5x-3x^2\right)}-\frac{18}{343}\int \frac{1}{-1-3x}\,dx+\frac{18}{343}\int \frac{1}{6-3x}\,dx\\ &= -\frac{5-6x}{49\left(2+5x-3x^2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(1+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{5-6x}{49\left(3x^2-5x-2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] (5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2+5x-3x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(2 + 5*x - 3*x^2)^(-2), x]
```

fricas [A] time = 0.38, size = 53, normalized size = 1.26

$$\frac{6\left(3x^2-5x-2\right)\log\left(3x+1\right)-6\left(3x^2-5x-2\right)\log\left(x-2\right)-42x+35}{343\left(3x^2-5x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) -
42*x + 35)/(3*x^2 - 5*x - 2)
```

giac [A] time = 0.32, size = 36, normalized size = 0.86

$$-\frac{6\,x-5}{49\,\left(3\,x^2-5\,x-2\right)}+\frac{6}{343}\,\log\left(|3\,x+1|\right)-\frac{6}{343}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(abs
(x - 2))
```

maple [A] time = 0.05, size = 32, normalized size = 0.76

$$\frac{6\ln\left(3x+1\right)}{343}-\frac{6\ln\left(x-2\right)}{343}-\frac{1}{49\left(x-2\right)}-\frac{3}{49\left(3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^2,x)
```

```
[Out] -1/49/(x-2)-6/343*ln(x-2)-3/49/(3*x+1)+6/343*ln(3*x+1)
```

maxima [A] time = 1.35, size = 34, normalized size = 0.81

$$-\frac{6\,x-5}{49\,\left(3\,x^2-5\,x-2\right)}+\frac{6}{343}\,\log\left(3\,x+1\right)-\frac{6}{343}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)
```

mupad [B] time = 0.08, size = 34, normalized size = 0.81

$$\frac{6\,\ln\left(\frac{3\,x+1}{x-2}\right)}{343}+\frac{2\,\left(3\,x-\frac{5}{2}\right)}{49\,\left(-3\,x^2+5\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2)^2,x)
```

```
[Out] (6*log((3*x + 1)/(x - 2)))/343 + (2*(3*x - 5/2))/(49*(5*x - 3*x^2 + 2))
```

sympy [A] time = 0.15, size = 32, normalized size = 0.76

$$\frac{5-6x}{147x^2-245x-98}-\frac{6\log(x-2)}{343}+\frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**2,x)
```

```
[Out] (5 - 6*x)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343
```

3.76 $\int \frac{1}{\left(a+cx+bx^2\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{2bx+c}{\left(4ab-c^2\right)\left(a+bx^2+cx\right)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {614, 618, 204}

$$\frac{2bx+c}{\left(4ab-c^2\right)\left(a+bx^2+cx\right)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+cx+bx^2\right)^2}\,dx &= \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)}+\frac{(2b)\int \frac{1}{a+cx+bx^2}\,dx}{4ab-c^2}\\
&= \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)}-\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-4ab+c^2-x^2}\,dx,x,c+2bx\right)}{4ab-c^2}\\
&= \frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)}+\frac{4b\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.99

$$
\frac{2bx+c}{\left(4ab-c^2\right)(a+x(bx+c))}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(a+cx+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x + b*x^2)^(-2), x]
```

fricas [B] time = 0.40, size = 334, normalized size = 4.70

$$
\left[\frac{4\,abc-c^3+2\left(b^2x^2+bcx+ab\right)\sqrt{-4\,ab+c^2}\,\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2+\sqrt{-4\,ab+c^2}\,(2\,bx+c)}{bx^2+cx+a}\right)+2\left(4\,ab^2-bc^2\right)x}{16\,a^3b^2-8\,a^2bc^2+ac^4+\left(16\,a^2b^3-8\,ab^2c^2+bc^4\right)x^2+\left(16\,a^2b^2c-8\,abc^3+c^5\right)x},\ \frac{4\,abc-c^3-4\left(b^2x^2+bcx+ab\right)\sqrt{4\,ab-c^2}\,\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)+2\left(4\,ab^2-bc^2\right)x}{16\,a^3b^2-8\,a^2bc^2+ac^4+\left(16\,a^2b^3-8\,ab^2c^2+bc^4\right)x^2+\left(16\,a^2b^2c-8\,abc^3+c^5\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")
```

```
[Out] [(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*x
^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x +
 a)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^2*b
^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x), (4*a*b
*c - c^3 - 4*(b^2*x^2 + b*c*x + a*b)*sqrt(4*a*b - c^2)*arctan(-(2*b*x + c)/
sqrt(4*a*b - c^2)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c
^4 + (16*a^2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c
^5)*x)]
```

giac [A] time = 0.40, size = 67, normalized size = 0.94

$$\frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\left(4\,ab-c^2\right)^{\frac{3}{2}}}+\frac{2\,bx+c}{\left(bx^2+cx+a\right)\left(4\,ab-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (2*b*x + c)
/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

maple [A] time = 0.06, size = 68, normalized size = 0.96

$$\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{\frac{3}{2}}}+\frac{2bx+c}{\left(4ab-c^2\right)\left(b\,x^2+cx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a)^2,x)
```

```
[Out] (2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))
/(4*a*b-c^2)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for mo
re details)Is c^2-4*a*b positive or negative?
```

mupad [B] time = 0.17, size = 119, normalized size = 1.68

$$\frac{\frac{c}{4\,a\,b-c^2}+\frac{2\,b\,x}{4\,a\,b-c^2}}{b\,x^2+c\,x+a}-\frac{4\,b\,\mathrm{atan}\left(\frac{\left(\frac{2\,b\,\left(c^3-4\,a\,b\,c\right)}{{\left(4\,a\,b-c^2\right)}^{5/2}}-\frac{4\,b^2\,x}{{\left(4\,a\,b-c^2\right)}^{3/2}}\right)\,\left(4\,a\,b-c^2\right)}{2\,b}\right)}{{\left(4\,a\,b-c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x + b*x^2)^2,x)
```

```
[Out] (c/(4*a*b - c^2) + (2*b*x)/(4*a*b - c^2))/(a + c*x + b*x^2) - (4*b*atan((((
2*b*(c^3 - 4*a*b*c))/(4*a*b - c^2)^(5/2) - (4*b^2*x)/(4*a*b - c^2)^(3/2))*(
4*a*b - c^2))/(2*b)))/(4*a*b - c^2)^(3/2)
```

sympy [B] time = 0.59, size = 265, normalized size = 3.73

$$-2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\log\left(x+\frac{-32a^2b^3\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}-2bc^4\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc}{4b^2}\right)+2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\log\left(x+\frac{32a^2b^3\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}-16ab^2c^2\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc^4\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc}{4b^2}\right)+\frac{2bx+c}{4a^2b-ac^2+x^2\left(4ab^2-bc^2\right)+x\left(4abc-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a)**2,x)
```

```
[Out] -2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4*a*b - c**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*b*c**4*sqrt(-1/(4*a*
b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (
32*a**2*b**3*sqrt(-1/(4*a*b - c**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b - c
**2)**3) + 2*b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2*b*x
+ c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**3))
```

3.77 $\int \frac{1}{\left(b+2ax+bx^2\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {614, 618, 206}

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a + b*x)/Sqrt[
a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(b+2ax+bx^2\right)^2}\,dx = -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} - \frac{b\int \frac{1}{b+2ax+bx^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right)-x^2}\,dx, x, 2a+2bx\right)}{a^2-b^2}$$

$$= -\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)} + \frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 1.00

$$\frac{a+bx}{2\left(b^2-a^2\right)\left(2ax+bx^2+b\right)} + \frac{b\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{2\left(b^2-a^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] (a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[-
a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b+2ax+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*a*x + b*x^2)^(-2), x]
```

fricas [B] time = 0.42, size = 317, normalized size = 4.40

$$\left[-\frac{2\,a^3-2\,ab^2+\left(b^2x^2+2\,abx+b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}\,(bx+a)}{bx^2+2\,ax+b}\right)+2\left(a^2b-b^3\right)x}{4\left(a^4b-2\,a^2b^3+b^5+\left(a^4b-2\,a^2b^3+b^5\right)x^2+2\left(a^5-2\,a^3b^2+ab^4\right)x\right)}, -\frac{a^3-ab^2-\left(b^2x^2+2\,abx+b^2\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(bx+a)}{a^2-b^2}\right)+\left(a^2b-b^3\right)x}{2\left(a^4b-2\,a^2b^3+b^5+\left(a^4b-2\,a^2b^3+b^5\right)x^2+2\left(a^5-2\,a^3b^2+ab^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^2
*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*x
+ b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 +
b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x), -1/2*(a^3 - a*b^2 - (b^2*x^2 + 2
*a*b*x + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2 - b^
2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)
*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x)]
```

giac [A] time = 0.51, size = 75, normalized size = 1.04

$$-\frac{b \arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{bx+a}{2\left(bx^2+2\,ax+b\right)\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) -
1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

maple [A] time = 0.05, size = 86, normalized size = 1.19

$$\frac{2b \arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\left(-4a^2+4b^2\right)\sqrt{-a^2+b^2}}+\frac{2bx+2a}{\left(-4a^2+4b^2\right)\left(b\,x^2+2ax+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(1
/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.32, size = 107, normalized size = 1.49

$$-\frac{\frac{a}{2\,\left(a^2-b^2\right)}+\frac{b\,x}{2\,\left(a^2-b^2\right)}}{b\,x^2+2\,a\,x+b}+\frac{b\,\mathrm{atan}\left(\frac{-a^3\,1\mathrm{i}-1\mathrm{i}\,x\,a^2\,b+a\,b^2\,1\mathrm{i}+1\mathrm{i}\,x\,b^3}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)\,1\mathrm{i}}{2\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x + b*x^2)^2,x)
```

```
[Out] (b*atan((a*b^2*1i + b^3*x*1i - a^3*1i - a^2*b*x*1i)/((a + b)^(3/2)*(a - b)^
(3/2)))*1i)/(2*(a + b)^(3/2)*(a - b)^(3/2)) - (a/(2*(a^2 - b^2)) + (b*x)/(2
*(a^2 - b^2)))/(b + 2*a*x + b*x^2)
```

sympy [B] time = 0.55, size = 230, normalized size = 3.19

$$-\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab-b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}-2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab+b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}+\frac{-a-bx}{2a^2b-2b^3+x^2\left(2a^2b-2b^3\right)+x\left(4a^3-4ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a - b)**3*(a +
 b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b - b**5*sqrt(1/
((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/((a - b)**3*(a + b)**3))*log(x
 + (a**4*b*sqrt(1/((a - b)**3*(a + b)**3)) - 2*a**2*b**3*sqrt(1/((a - b)**3
*(a + b)**3)) + a*b + b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 + (-a -
 b*x)/(2*a**2*b - 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a*b**2)
)
```

3.78 $\int \frac{1}{\left(b+2ax-bx^2\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{a-bx}{2\left(a^2+b^2\right)\left(2ax-bx^2+b\right)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {614, 618, 206}

$$-\frac{a-bx}{2\left(a^2+b^2\right)\left(2ax-bx^2+b\right)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - b*x)/Sqrt[
a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(b+2ax-bx^2\right)^2}\,dx = -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)}+\frac{b\int \frac{1}{b+2ax-bx^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)}-\frac{b\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2a-2bx\right)}{a^2+b^2}$$

$$= -\frac{a-bx}{2\left(a^2+b^2\right)\left(b+2ax-bx^2\right)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.13

$$\frac{\frac{bx-a}{2ax-bx^2+b}-\frac{b\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-2),x]
[Out] ((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b+2ax-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*a*x - b*x^2)^(-2),x]
[Out] IntegrateAlgebraic[(b + 2*a*x - b*x^2)^(-2), x]
```

fricas [B] time = 0.41, size = 171, normalized size = 2.48

$$-\frac{2\,a^3+2\,ab^2+\left(b^2x^2-2\,abx-b^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}\,(bx-a)}{bx^2-2\,ax-b}\right)-2\left(a^2b+b^3\right)x}{4\left(a^4b+2\,a^2b^3+b^5-\left(a^4b+2\,a^2b^3+b^5\right)x^2+2\left(a^5+2\,a^3b^2+ab^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2*
x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x -
 b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b
^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)
```

giac [A] time = 0.53, size = 90, normalized size = 1.30

$$-\frac{b\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{bx-a}{2\left(bx^2-2\,ax-b\right)\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^
2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 + b^
2))
```

maple [A] time = 0.05, size = 84, normalized size = 1.22

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{2bx-2a}{2\sqrt{a^2+b^2}}\right)}{\left(-4a^2-4b^2\right)\sqrt{a^2+b^2}}+\frac{2bx-2a}{\left(-4a^2-4b^2\right)\left(b\,x^2-2ax-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x-2*a)/(-4*a^2-4*b^2)/(b*x^2-2*a*x-b)-2*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 2.84, size = 97, normalized size = 1.41

$$-\frac{b\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{4\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{bx-a}{2\left(a^2b+b^3-\left(a^2b+b^3\right)x^2+2\left(a^3+ab^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/(a^2 +
b^2)^(3/2) + 1/2*(b*x - a)/(a^2*b + b^3 - (a^2*b + b^3)*x^2 + 2*(a^3 + a*b^
2)*x)
```

mupad [B] time = 0.32, size = 100, normalized size = 1.45

$$-\frac{\frac{a}{2\left(a^2+b^2\right)}-\frac{b\,x}{2\left(a^2+b^2\right)}}{-b\,x^2+2\,a\,x+b}+\frac{b\,\mathrm{atan}\left(\frac{a\,b^2\,1\mathrm{i}+a^3\,1\mathrm{i}-b\,x\,\left(a^2+b^2\right)\,1\mathrm{i}}{\left(a^2+b^2\right)^{3/2}}\right)\,1\mathrm{i}}{2\,\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x - b*x^2)^2,x)
```

```
[Out] (b*atan((a*b^2*1i + a^3*1i - b*x*(a^2 + b^2)*1i)/(a^2 + b^2)^(3/2))*1i)/(2*
(a^2 + b^2)^(3/2)) - (a/(2*(a^2 + b^2)) - (b*x)/(2*(a^2 + b^2)))/(b + 2*a*x
 - b*x^2)
```

sympy [B] time = 0.60, size = 218, normalized size = 3.16

$$-\frac{b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-2a^2b^3\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-ab-b^5\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}+2a^2b^3\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}-ab+b^5\sqrt{\frac{1}{\left(a^2+b^2\right)^3}}}{b^2}\right)}{4}+\frac{a-bx}{-2a^2b-2b^3+x^2\left(2a^2b+2b^3\right)+x\left(-4a^3-4ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)**(-3)) - 2
*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt((a**2 + b**2)**(-3))
)/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x + (a**4*b*sqrt((a**2 + b**2)*
*(-3)) + 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b + b**5*sqrt((a**2 + b*
*2)**(-3)))/b**2)/4 + (a - b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b + 2*b*
*3) + x*(-4*a**3 - 4*a*b**2))
```

3.79 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

Optimal. Leaf size=62

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

Rubi [A] time = 0.16, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {618, 204}

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),x]
```

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Csc
[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\left(\frac{a}{b}\right)^{2/n}\left(1-\cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)}\,dx, x, 2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\text{co}\right.\right.$$

$$= -\left(\frac{a}{b}\right)^{-1/n}\tan^{-1}\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-\right.$$

Mathematica [A] time = 0.10, size = 65, normalized size = 1.05

$$\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\frac{\tan\left(\frac{\pi-2\pi k}{2n}\right)\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}+x\right)}{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),
x]
```

```
[Out] (ArcTan[(((a/b)^n^(-1) + x)*Tan[(Pi - 2*k*Pi)/(2*n)])/((a/b)^n^(-1) - x)]*C
sc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/
n])^(-1),x]
```

```
[Out] IntegrateAlgebraic[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/
n])^(-1), x]
```

fricas [A] time = 0.42, size = 89, normalized size = 1.44

$$-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="fricas")
```

```
[Out] -arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n -
pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

giac [A] time = 0.57, size = 100, normalized size = 1.61

$$\frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(-\frac{2\pi k}{n}+\frac{\pi}{n}\right)-x}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^2+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="giac")
```

```
[Out] arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n)
^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

maple [A] time = 0.57, size = 111, normalized size = 1.79

$$\frac{\arctan\left(\frac{-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)+2x}{2\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}\right)}{\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x)
```

```
[Out] 1/(-cos(Pi*(2*k-1)/n)^2*((a/b)^(1/n))^2+(a/b)^(2/n))^(1/2)*arctan(1/2*(2*x-
2*cos(Pi*(2*k-1)/n)*(a/b)^(1/n))/(-cos(Pi*(2*k-1)/n)^2*((a/b)^(1/n))^2+(a/b
)^(2/n))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1>0)', see `assume?` for more detai
ls)Is 1 zero or nonzero?
```

mupad [B] time = 0.25, size = 110, normalized size = 1.77

$$-\frac{\operatorname{atanh}\left(\frac{x-\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)\,\left(\frac{a}{b}\right)^{1/n}}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}\right)}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/b)^(2/n) + x^2 - 2*x*cos((Pi - 2*Pi*k)/n)*(a/b)^(1/n)),x)
```

```
[Out] -atanh((x - cos((Pi*(2*k - 1))/n)*(a/b)^(1/n))/((cos((Pi*(2*k - 1))/n) - 1)
^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n)))/((cos((Pi*(2*k - 1))
/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n))
```

sympy [B] time = 0.98, size = 212, normalized size = 3.42

$$-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^{2}\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^{2}\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

```
[Out] -sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(
2*pi*k/n - pi/n) - sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/
b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt((a/b)**(-2/
n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(2*pi*k/n - pi/n)
+ sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*p
i*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2
```

3.80 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{b^2-4ab^3}}{b}\right)}{b}$$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {616, 31}

$$\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b - Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ab+\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx &= -\left(b\int \frac{1}{\frac{1}{2}\left(-b+\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\\ &= -\frac{\log\left(b+\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b-\sqrt{b^2-4ab^3}+2b^2x\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
[Out] (2*ArcTanh[(-Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1), x]
```

fricas [B] time = 0.41, size = 63, normalized size = 1.91

$$\frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
[Out] (log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*a
*b^3 + b^2))/b))/b
```

giac [A] time = 0.53, size = 56, normalized size = 1.70

$$-\frac{\log\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
[Out] -log(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

maple [A] time = 0.08, size = 31, normalized size = 0.94

$$-\frac{2\,\mathrm{arctanh}\left(\frac{-2b^{2}x+\sqrt{-(4ab-1)b^{2}}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] -2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

maxima [A] time = 1.39, size = 55, normalized size = 1.67

$$-\frac{\log\left(\frac{2\,b^{2}x-b-\sqrt{-4\,ab^{3}+b^{2}}}{2\,b^{2}x+b-\sqrt{-4\,ab^{3}+b^{2}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x - b - sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b - sqrt(-4*a*b^3 + b^
2)))/b
```

mupad [B] time = 0.26, size = 38, normalized size = 1.15

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}-\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b + x*(b^2 - 4*a*b^3)^(1/2) - b^2*x^2),x)
```

```
[Out] -(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) - (2*b^2*x)/(b^2)^(1/2)))/(b^2)
^(1/2)
```

sympy [B] time = 0.29, size = 56, normalized size = 1.70

$$-\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^{3}+b^{2}}}{2b^{2}}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^{3}+b^{2}}}{2b^{2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) - sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

3.81 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {616, 31}

$$\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b + Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ab-\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx &= -\left(b\int \frac{1}{\frac{1}{2}\left(-b-\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\\ &= -\frac{\log\left(b-\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b+\sqrt{b^2-4ab^3}+2b^2x\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-b^2(4ab-1)}+2b^2x}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
[Out] (2*ArcTanh[(Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1), x]
```

fricas [B] time = 0.42, size = 59, normalized size = 1.90

$$\frac{\log\left(\frac{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
[Out] (log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*a
*b^3 + b^2))/b))/b
```

giac [A] time = 0.55, size = 54, normalized size = 1.74

$$-\frac{\log\left(\frac{\left|2\,b^2x+\sqrt{-4\,ab+1}\,|b|-|b|\right|}{\left|2\,b^2x+\sqrt{-4\,ab+1}\,|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
[Out] -log(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

maple [A] time = 0.05, size = 31, normalized size = 1.00

$$\frac{2\,\mathrm{arctanh}\left(\frac{2b^2x+\sqrt{-(4ab-1)b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] 2/b*arctanh((2*b^2*x+(-(4*a*b-1)*b^2)^(1/2))/b)
```

maxima [A] time = 1.32, size = 51, normalized size = 1.65

$$-\frac{\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x - b + sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b + sqrt(-4*a*b^3 + b^
2)))/b
```

mupad [B] time = 0.13, size = 38, normalized size = 1.23

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}+\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(b^2 - 4*a*b^3)^(1/2) - a*b + b^2*x^2),x)
```

```
[Out] (2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) + (2*b^2*x)/(b^2)^(1/2)))/(b^2)^
(1/2)
```

sympy [B] time = 0.28, size = 56, normalized size = 1.81

$$-\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) + sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

3.82 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$

Optimal. Leaf size=17

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {618, 204}

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{1}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{1}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.12

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\frac{(x-1)\tan\left(\frac{1}{14}\right)}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
[Out] ArcTan[((-1 + x)*Tan[1/14])/(1 + x)]*Csc[1/7]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
[Out] IntegrateAlgebraic[(1 + x^2 + 2*x*Cos[1/7])^(-1), x]
```

fricas [A] time = 0.42, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")
[Out] arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

giac [B] time = 0.47, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

maple [B] time = 0.24, size = 33, normalized size = 1.94

$$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{1}{7}\right)}{2\sqrt{1-\left(\cos^{2}\left(\frac{1}{7}\right)\right)}}\right)}{\sqrt{1-\left(\cos^{2}\left(\frac{1}{7}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7)),x)
```

```
[Out] 1/(1-cos(1/7)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(1-cos(1/7)^2)^(1/2))
```

maxima [B] time = 2.95, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

mupad [B] time = 0.12, size = 27, normalized size = 1.59

$$\frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^{2}}}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x*cos(1/7) + x^2 + 1),x)
```

```
[Out] atan((x + cos(1/7))/(1 - cos(1/7)^2)^(1/2))/(1 - cos(1/7)^2)^(1/2)
```

sympy [C] time = 0.19, size = 165, normalized size = 9.71

$$-\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

```
[Out] -I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/7
)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(co
s(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sqrt(
cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos
(1/7))*sqrt(cos(1/7) + 1))
```

3.83 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

Optimal. Leaf size=23

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(x\csc\left(\frac{\pi}{7}\right)+\cot\left(\frac{\pi}{7}\right)\right)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {618, 204}

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{7}\right)\left(x+\cos\left(\frac{\pi}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Csc[Pi/7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{\pi}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{\pi}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\end{aligned}$$

Mathematica [B] time = 0.04, size = 56, normalized size = 2.43

$$\frac{2\tan^{-1}\left(\frac{2x-(-1)^{6/7}+\sqrt[7]{-1}}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right)}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(1/7) - (-1)^(6/7) + 2*x)/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
])/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2 + 2*x*Cos[Pi/7])^(-1), x]
```

fricas [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sin\left(\frac{1}{7}\,\pi\right)}\right)}{\sin\left(\frac{1}{7}\,\pi\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

giac [A] time = 0.47, size = 33, normalized size = 1.43

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

maple [B] time = 0.20, size = 39, normalized size = 1.70

$$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{\pi}{7}\right)}{2\sqrt{1-\left(\cos^{2}\left(\frac{\pi}{7}\right)\right)}}\right)}{\sqrt{1-\left(\cos^{2}\left(\frac{\pi}{7}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7*Pi)),x)
```

```
[Out] 1/(1-cos(1/7*Pi)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(1-cos(1/7*Pi)^2)^
(1/2))
```

maxima [A] time = 3.02, size = 33, normalized size = 1.43

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

mupad [B] time = 0.30, size = 42, normalized size = 1.83

$$-\frac{\operatorname{atanh}\left(\frac{x+\cos\left(\frac{\Pi}{7}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x*cos(Pi/7) + 1),x)
```

```
[Out] -atanh((x + cos(Pi/7))/((cos(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2)))/((cos
(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2))
```

sympy [C] time = 0.59, size = 70, normalized size = 3.04

$$-\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(2-2\cos^{2}\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}+\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(2-2\cos^{2}\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

```
[Out] -I*log(x + cos(pi/7) - I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
+ I*log(x + cos(pi/7) + I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
```

3.84 $\int \sqrt{5-6x+9x^2}\,dx$

Optimal. Leaf size=38

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 215}

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{5-6x+9x^2}\,dx &= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+2\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx\\ &= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)\\ &= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.03

$$\sqrt{9x^2-6x+5}\left(\frac{x}{2}-\frac{1}{6}\right)+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] (-1/6 + x/2)*Sqrt[5 - 6*x + 9*x^2] + (2*ArcSinh[(-1 + 3*x)/2])/3
```

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 1.26

$$\frac{1}{6}(3x-1)\sqrt{9x^2-6x+5}-\frac{2}{3}\log\left(\sqrt{9x^2-6x+5}-3x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ((-1 + 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 - (2*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^
2]])/3
```

fricas [A] time = 0.38, size = 40, normalized size = 1.05

$$\frac{1}{6}\sqrt{9\,x^2-6\,x+5}\,(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

giac [A] time = 0.47, size = 40, normalized size = 1.05

$$\frac{1}{6}\sqrt{9\,x^2-6\,x+5}\,(3\,x-1)-\frac{2}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

maple [A] time = 0.06, size = 29, normalized size = 0.76

$$\frac{2\operatorname{arcsinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}+\frac{(18x-6)\sqrt{9x^2-6x+5}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-6*x+5)^(1/2),x)
```

```
[Out] 1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

maxima [A] time = 3.04, size = 38, normalized size = 1.00

$$\frac{1}{2}\sqrt{9\,x^2-6\,x+5}\,x-\frac{1}{6}\sqrt{9\,x^2-6\,x+5}+\frac{2}{3}\operatorname{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*x
 - 1/2)
```

mupad [B] time = 0.08, size = 39, normalized size = 1.03

$$\frac{2\,\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}+\left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9\,x^2-6\,x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2 - 6*x + 5)^(1/2),x)
```

```
[Out] (2*log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1))/3 + (x/2 - 1/6)*(9*x^2 - 6*x + 5
)^(1/2)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{9x^2 - 6x + 5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 - 6*x + 5), x)
```

3.85 $\int \sqrt{3-4x-4x^2}\, dx$

Optimal. Leaf size=30

$$\frac{1}{4}\sqrt{-4x^2-4x+3}\,(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$\frac{1}{4}\sqrt{-4x^2-4x+3}\,(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + 2\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$$
$$= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} - \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -4-8x\right)$$
$$= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + \sin^{-1}\left(\frac{1}{2}+x\right)$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{1}{4}\sqrt{-4x^2-4x+3}\,(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*x - 4*x^2],x]
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

IntegrateAlgebraic [A] time = 0.09, size = 49, normalized size = 1.63

$$\frac{1}{4}(2x+1)\sqrt{-4x^2-4x+3} - 2\tan^{-1}\left(\frac{\sqrt{-4x^2-4x+3}}{2x+3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - 4*x - 4*x^2],x]
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - 2*ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2
*x)]
```

fricas [B] time = 0.41, size = 53, normalized size = 1.77

$$\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)-\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x +
 1)/(4*x^2 + 4*x - 3))
```

giac [A] time = 1.60, size = 24, normalized size = 0.80

$$\frac{1}{4}\,\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)+\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\arcsin\left(x+\frac{1}{2}\right)-\frac{(-8x-4)\sqrt{-4x^2-4x+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-4*x+3)^(1/2),x)
```

```
[Out] -1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(x+1/2)
```

maxima [A] time = 3.00, size = 38, normalized size = 1.27

$$\frac{1}{2}\,\sqrt{-4\,x^2-4\,x+3}\,x+\frac{1}{4}\,\sqrt{-4\,x^2-4\,x+3}-\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/2
)
```

mupad [B] time = 0.05, size = 23, normalized size = 0.77

$$\mathrm{asin}\left(x+\frac{1}{2}\right)+\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{-4\,x^2-4\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*x^2 - 4*x)^(1/2),x)
```

```
[Out] asin(x + 1/2) + (x/2 + 1/4)*(3 - 4*x^2 - 4*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4x^2-4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 - 4*x + 3), x)
```

3.86 $\int \sqrt{-8+6x+9x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x +
 9*x^2]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - \frac{9}{2}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$$
$$= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - 9\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)$$
$$= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - \frac{3}{2}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8} - \frac{3}{2}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
[Out] (1/6 + x/2)*Sqrt[-8 + 6*x + 9*x^2] - (3*Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2
]])/2
```

IntegrateAlgebraic [A] time = 0.11, size = 49, normalized size = 1.00

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8} - 3\tanh^{-1}\left(\frac{\sqrt{9x^2+6x-8}}{3x-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-8 + 6*x + 9*x^2],x]
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - 3*ArcTanh[Sqrt[-8 + 6*x + 9*x^2]/(-2
 + 3*x)]
```

fricas [A] time = 0.38, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{9\,x^2+6\,x-8}\,(3\,x+1) + \frac{3}{2}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8)
- 1)
```

giac [A] time = 0.43, size = 41, normalized size = 0.84

$$\frac{1}{6}\sqrt{9x^2+6x-8}(3x+1)+\frac{3}{2}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x -
 8) - 1))
```

maple [A] time = 0.05, size = 50, normalized size = 1.02

$$-\frac{\sqrt{9}\ \ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)}{2}+\frac{(18x+6)\sqrt{9x^2+6x-8}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/36*(18*x+6)*(9*x^2+6*x-8)^(1/2)-1/2*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^
(1/2))*9^(1/2)
```

maxima [A] time = 2.87, size = 52, normalized size = 1.06

$$\frac{1}{2}\sqrt{9x^2+6x-8}\,x+\frac{1}{6}\sqrt{9x^2+6x-8}-\frac{3}{2}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6*
sqrt(9*x^2 + 6*x - 8) + 6)
```

mupad [B] time = 0.21, size = 39, normalized size = 0.80

$$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3\ln\left(3x+\sqrt{9x^2+6x-8}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x + 9*x^2 - 8)^(1/2),x)
```

```
[Out] (x/2 + 1/6)*(6*x + 9*x^2 - 8)^(1/2) - (3*log(3*x + (6*x + 9*x^2 - 8)^(1/2)
+ 1))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{9x^2 + 6x - 8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+6*x-8)**(1/2),x)
[Out] Integral(sqrt(9*x**2 + 6*x - 8), x)
```

3.87 $\int \sqrt{2+4x+3x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 215}

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3]
)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{1}{3}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$$
$$= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx, x, 4+6x\right)}{6\sqrt{6}}$$
$$= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\sqrt{3x^2+4x+2}\left(\frac{x}{2}+\frac{1}{3}\right)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x + 3*x^2],x]
[Out] (1/3 + x/2)*Sqrt[2 + 4*x + 3*x^2] + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.13, size = 59, normalized size = 1.31

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x+2}-\frac{\log\left(\sqrt{3}\sqrt{3x^2+4x+2}-3x-2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + 4*x + 3*x^2],x]
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 - Log[-2 - 3*x + Sqrt[3]*Sqrt[2 + 4*x +
 3*x^2]]/(3*Sqrt[3])
```

fricas [A] time = 0.40, size = 58, normalized size = 1.29

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+4x+2}(3x+2)-9x^2-12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
```

giac [A] time = 0.46, size = 53, normalized size = 1.18

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2)-\frac{1}{9}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x -
 sqrt(3*x^2 + 4*x + 2)) - 2)
```

maple [A] time = 0.05, size = 35, normalized size = 0.78

$$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9}+\frac{(6x+4)\sqrt{3x^2+4x+2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/12*(6*x+4)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

maxima [A] time = 3.00, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{3x^2+4x+2}\,x+\frac{1}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}\,(3x+2)\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) +
1/3*sqrt(3*x^2 + 4*x + 2)
```

mupad [B] time = 0.19, size = 48, normalized size = 1.07

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3x^2+4x+2}+\frac{\sqrt{3}\,(3x+2)}{3}\right)}{9}+\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*log((4*x + 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x + 2))/3))/9 + (x/2 + 1
/3)*(4*x + 3*x^2 + 2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 4x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x + 2), x)
```

3.88 $\int \sqrt{2+4x-3x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}\,(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}\,(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sq
rt[3])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+4x-3x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} + \frac{5}{3}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$$
$$= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{1}{6}\sqrt{\frac{5}{6}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx, x, 4-6x\right)$$
$$= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2} - \frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] (-1/3 + x/2)*Sqrt[2 + 4*x - 3*x^2] - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sqrt
[3])
```

IntegrateAlgebraic [A] time = 0.25, size = 73, normalized size = 1.62

$$\frac{1}{6}\sqrt{-3x^2+4x+2}\,(3x-2) + \frac{10\tan^{-1}\left(\frac{\sqrt{3}x-\sqrt{\frac{10}{3}}-\frac{2}{\sqrt{3}}}{\sqrt{-3x^2+4x+2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 + (10*ArcTan[(-2/Sqrt[3] - Sqrt[10/3]
+ Sqrt[3]*x)/Sqrt[2 + 4*x - 3*x^2]])/(3*Sqrt[3])
```

fricas [A] time = 0.39, size = 60, normalized size = 1.33

$$\frac{1}{6}\sqrt{-3\,x^2+4\,x+2}\,(3\,x-2) - \frac{5}{9}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4\,x+2}\,(3\,x-2)}{3\left(3\,x^2-4\,x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(
-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))
```

giac [A] time = 0.47, size = 36, normalized size = 0.80

$$\frac{1}{6}\sqrt{-3\,x^2+4\,x+2}\,(3\,x-2)+\frac{5}{9}\sqrt{3}\,\arcsin\left(\frac{1}{10}\sqrt{10}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*
x - 2))
```

maple [A] time = 0.05, size = 35, normalized size = 0.78

$$\frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{9}-\frac{(-6x+4)\sqrt{-3x^2+4x+2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3
))
```

maxima [A] time = 2.97, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{-3\,x^2+4\,x+2}\,x-\frac{5}{9}\sqrt{3}\,\arcsin\left(-\frac{1}{10}\sqrt{10}\,(3\,x-2)\right)-\frac{1}{3}\sqrt{-3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.78

$$\frac{5\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{10}\,(3\,x-2)}{10}\right)}{9}+\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asin((10^(1/2)*(3*x - 2))/10))/9 + (x/2 - 1/3)*(4*x - 3*x^2 + 2)
^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 4x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x + 2), x)
```

3.89 $\int \sqrt{2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
+ 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{24}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$$
$$= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}}\right)$$
$$= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{24\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x+2} - \sqrt{3}\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*Log[5 + 6*x + 2*Sqrt[6 + 15*x
+ 9*x^2]])/72
```

IntegrateAlgebraic [A] time = 0.18, size = 59, normalized size = 0.95

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2} - \frac{\tanh^{-1}\left(\frac{\sqrt{3x^2+5x+2}}{\sqrt{3}(x+1)}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[Sqrt[2 + 5*x + 3*x^2]/(Sqrt[
3]*(1 + x))]/(12*Sqrt[3])
```

fricas [A] time = 0.40, size = 58, normalized size = 0.94

$$\frac{1}{12}\sqrt{3\,x^2+5\,x+2}\,(6\,x+5) + \frac{1}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2+5\,x+2}\,(6\,x+5)+72\,x^2+120\,x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3*
x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)
```

giac [A] time = 0.44, size = 54, normalized size = 0.87

$$\frac{1}{12}\sqrt{3x^2+5x+2}\,(6x+5)+\frac{1}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

maple [A] time = 0.05, size = 50, normalized size = 0.81

$$-\frac{\sqrt{3}\ \ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)}{72}+\frac{(6x+5)\sqrt{3x^2+5x+2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2
)^(1/2))*3^(1/2)
```

maxima [A] time = 2.98, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2+5x+2}\,x-\frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x +
 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

mupad [B] time = 0.20, size = 48, normalized size = 0.77

$$\left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x+2}-\frac{\sqrt{3}\ \ln\left(\sqrt{3x^2+5x+2}+\frac{\sqrt{3}\left(3x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (x/2 + 5/12)*(5*x + 3*x^2 + 2)^(1/2) - (3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2)
 + (3^(1/2)*(3*x + 5/2))/3))/72
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 5x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x + 2), x)
```

3.90 $\int \sqrt{2+5x-3x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}\,(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}\,(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3
])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} + \frac{49}{24}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$$
$$= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx, x, 5-6x\right)}{24\sqrt{3}}$$
$$= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x+2} - \frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x - 3*x^2],x]
[Out] (-5/12 + x/2)*Sqrt[2 + 5*x - 3*x^2] - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.13, size = 61, normalized size = 1.42

$$\frac{1}{12}(6x-5)\sqrt{-3x^2+5x+2} - \frac{49\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}}{3x+1}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + 5*x - 3*x^2],x]
[Out] ((-5 + 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcTan[(Sqrt[3]*Sqrt[2 + 5*x -
3*x^2])/(1 + 3*x)])/(12*Sqrt[3])
```

fricas [A] time = 0.41, size = 60, normalized size = 1.40

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5) - \frac{49}{72}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)}{6\left(3\,x^2-5\,x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*sq
rt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))
```

giac [A] time = 0.40, size = 31, normalized size = 0.72

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)+\frac{49}{72}\,\sqrt{3}\,\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

maple [A] time = 0.04, size = 32, normalized size = 0.74

$$\frac{49\sqrt{3}\ \arcsin\left(\frac{6x}{7}-\frac{5}{7}\right)}{72}-\frac{(-6x+5)\sqrt{-3x^2+5x+2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

maxima [A] time = 3.08, size = 41, normalized size = 0.95

$$\frac{1}{2}\sqrt{-3\,x^2+5\,x+2}\,x-\frac{49}{72}\,\sqrt{3}\,\arcsin\left(-\frac{6}{7}\,x+\frac{5}{7}\right)-\frac{5}{12}\,\sqrt{-3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7) - 5/12*sq
rt(-3*x^2 + 5*x + 2)
```

mupad [B] time = 0.15, size = 30, normalized size = 0.70

$$\frac{49\,\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\,\sqrt{-3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (49*3^(1/2)*asin((6*x)/7 - 5/7))/72 + (x/2 - 5/12)*(5*x - 3*x^2 + 2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 5x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x + 2), x)
```

3.91 $\int \sqrt{-2+4x+3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5}{3}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$$
$$= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{10}{3}\,\mathrm{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)$$
$$= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.90

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2} - \frac{5\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*Log[2 + 3*x + Sqrt[-6 + 12*x + 9*
x^2]])/(3*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.31, size = 66, normalized size = 1.12

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2} - \frac{10\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{3x^2+4x-2}}{3x+\sqrt{10}+2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (10*ArcTanh[(Sqrt[3]*Sqrt[-2 + 4*x +
 3*x^2])/(2 + Sqrt[10] + 3*x)])/(3*Sqrt[3])
```

fricas [A] time = 0.39, size = 58, normalized size = 0.98

$$\frac{1}{6}\sqrt{3\,x^2+4\,x-2}\,(3\,x+2) + \frac{5}{18}\sqrt{3}\,\log\left(-\sqrt{3}\sqrt{3\,x^2+4\,x-2}\,(3\,x+2)+9\,x^2+12\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

giac [A] time = 0.49, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{3x^2+4x-2}\,(3x+2)+\frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

maple [A] time = 0.05, size = 50, normalized size = 0.85

$$-\frac{5\sqrt{3}\,\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)}{9}+\frac{(6x+4)\sqrt{3x^2+4x-2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/12*(6*x+4)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^(
1/2))*3^(1/2)
```

maxima [A] time = 2.98, size = 58, normalized size = 0.98

$$\frac{1}{2}\sqrt{3x^2+4x-2}\,x-\frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)+\frac{1}{3}\sqrt{3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x -
2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

mupad [B] time = 0.19, size = 48, normalized size = 0.81

$$\left(\frac{x}{2}+\frac{1}{3}\right)\sqrt{3x^2+4x-2}-\frac{5\sqrt{3}\,\ln\left(\sqrt{3x^2+4x-2}+\frac{\sqrt{3}\,(3x+2)}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 - 2)^(1/2),x)
```

```
[Out] (x/2 + 1/3)*(4*x + 3*x^2 - 2)^(1/2) - (5*3^(1/2)*log((4*x + 3*x^2 - 2)^(1/2
) + (3^(1/2)*(3*x + 2))/3))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 4x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x - 2), x)
```

3.92 $\int \sqrt{-2+4x-3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 204}

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 +
 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-2+4x-3x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{1}{3}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$$
$$= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)$$
$$= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{-3x^2+4x-2}\,(3x-2) + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9
*x^2]]/(3*Sqrt[3])
```

IntegrateAlgebraic [C] time = 0.15, size = 65, normalized size = 1.10

$$\frac{1}{6}(3x-2)\sqrt{-3x^2+4x-2} - \frac{i\log\left(\sqrt{3}\sqrt{-3x^2+4x-2} - 3ix + 2i\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 - ((I/3)*Log[2*I - (3*I)*x + Sqrt[3]*
Sqrt[-2 + 4*x - 3*x^2]])/Sqrt[3]
```

fricas [C] time = 0.40, size = 84, normalized size = 1.42

$$\frac{1}{6}\sqrt{-3x^2+4x-2}\,(3x-2) + \frac{1}{18}i\sqrt{3}\log\left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2} - 6x + 4}{x}\right) - \frac{1}{18}i\sqrt{3}\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2} - 6x + 4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) + 1/18*I*sqrt(3)*log((2*I*sqrt(3)*sqrt
(-3*x^2 + 4*x - 2) - 6*x + 4)/x) - 1/18*I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3
*x^2 + 4*x - 2) - 6*x + 4)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3\,x^2+4\,x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*x^2 + 4*x - 2), x)
```

maple [A] time = 0.05, size = 46, normalized size = 0.78

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9}-\frac{(-6x+4)\sqrt{-3x^2+4x-2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*
x^2+4*x-2)^(1/2))
```

maxima [C] time = 2.89, size = 46, normalized size = 0.78

$$\frac{1}{2}\sqrt{-3\,x^2+4\,x-2}\,x+\frac{1}{9}i\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{2}\,(3\,x-2)\right)-\frac{1}{3}\,\sqrt{-3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

mupad [B] time = 0.05, size = 36, normalized size = 0.61

$$\frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{2}\,(3\,x-2)\,1{}\mathrm{i}}{2}\right)}{9}+\left(\frac{x}{2}-\frac{1}{3}\right)\,\sqrt{-3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin((2^(1/2)*(3*x - 2)*1i)/2))/9 + (x/2 - 1/3)*(4*x - 3*x^2 - 2)^
(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 4x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x - 2), x)
```

3.93 $\int \sqrt{-2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sq
rt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-2+5x+3x^2}\,dx &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49}{24}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx \\ &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49}{12}\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}}\right) \\ &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x-2} - 49\sqrt{3}\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*Log[5 + 6*x + 2*Sqrt[-6 +
15*x + 9*x^2]])/72
```

IntegrateAlgebraic [A] time = 0.18, size = 59, normalized size = 0.95

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2} - \frac{49\tanh^{-1}\left(\frac{\sqrt{3x^2+5x-2}}{\sqrt{3}(x+2)}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[Sqrt[-2 + 5*x + 3*x^2]/
(Sqrt[3]*(2 + x))])/(12*Sqrt[3])
```

fricas [A] time = 0.40, size = 58, normalized size = 0.94

$$\frac{1}{12}\sqrt{3x^2+5x-2}\,(6x+5) + \frac{49}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2+5x-2}\,(6x+5)+72x^2+120x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3
*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)
```

giac [A] time = 0.44, size = 54, normalized size = 0.87

$$\frac{1}{12}\sqrt{3x^2+5x-2}\,(6x+5)+\frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

maple [A] time = 0.05, size = 50, normalized size = 0.81

$$-\frac{49\sqrt{3}\,\ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)}{72}+\frac{(6x+5)\sqrt{3x^2+5x-2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/12*(6*x+5)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x-
2)^(1/2))*3^(1/2)
```

maxima [A] time = 3.00, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2+5x-2}\,x-\frac{49}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
- 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

mupad [B] time = 0.22, size = 48, normalized size = 0.77

$$\left(\frac{x}{2}+\frac{5}{12}\right)\sqrt{3x^2+5x-2}-\frac{49\sqrt{3}\,\ln\left(\sqrt{3x^2+5x-2}+\frac{\sqrt{3}\left(3x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 - 2)^(1/2),x)
```

```
[Out] (x/2 + 5/12)*(5*x + 3*x^2 - 2)^(1/2) - (49*3^(1/2)*log((5*x + 3*x^2 - 2)^(1
/2) + (3^(1/2)*(3*x + 5/2))/3))/72
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 5x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x - 2), x)
```

3.94 $\int \sqrt{-2+5x-3x^2}\, dx$

Optimal. Leaf size=39

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}\,(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}\,(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{-2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} + \frac{1}{24}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$$
$$= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-6x\right)}{24\sqrt{3}}$$
$$= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x-2} - \frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
[Out] (-5/12 + x/2)*Sqrt[-2 + 5*x - 3*x^2] - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.13, size = 61, normalized size = 1.56

$$\frac{1}{12}(6x-5)\sqrt{-3x^2+5x-2} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-3x^2+5x-2}}{3x-2}\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-2 + 5*x - 3*x^2],x]
[Out] ((-5 + 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcTan[(Sqrt[3]*Sqrt[-2 + 5*x - 3*
x^2])/(-2 + 3*x)]/(12*Sqrt[3])
```

fricas [A] time = 0.41, size = 60, normalized size = 1.54

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5) - \frac{1}{72}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)}{6\left(3\,x^2-5\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sqr
t(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))
```

giac [A] time = 0.53, size = 31, normalized size = 0.79

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)+\frac{1}{72}\sqrt{3}\,\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{\sqrt{3}\,\arcsin\left(6x-5\right)}{72}-\frac{\left(-6x+5\right)\sqrt{-3x^2+5x-2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x-2)^(1/2),x)
[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(-6*x+5)*(-3*x^2+5*x-2)^(1/2)
```

maxima [A] time = 3.08, size = 41, normalized size = 1.05

$$\frac{1}{2}\sqrt{-3\,x^2+5\,x-2}\,x+\frac{1}{72}\sqrt{3}\,\arcsin\left(6\,x-5\right)-\frac{5}{12}\sqrt{-3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3*
x^2 + 5*x - 2)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.77

$$\frac{\sqrt{3}\,\mathrm{asin}\left(6\,x-5\right)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*asin(6*x - 5))/72 + (x/2 - 5/12)*(5*x - 3*x^2 - 2)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 5x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x - 2), x)
```

3.95 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 215}

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx &= \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)\\ &= \frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

IntegrateAlgebraic [A] time = 0.09, size = 24, normalized size = 1.71

$$-\frac{1}{3}\log\left(\sqrt{9x^2-6x+5}-3x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[5 - 6*x + 9*x^2],x]
[Out] -1/3*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^2]]
```

fricas [B] time = 0.40, size = 20, normalized size = 1.43

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

giac [B] time = 0.63, size = 20, normalized size = 1.43

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="giac")
[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.64

$$\frac{\operatorname{arcsinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-6*x+5)^(1/2),x)
[Out] 1/3*arcsinh(3/2*x-1/2)
```

maxima [A] time = 2.95, size = 8, normalized size = 0.57

$$\frac{1}{3}\,\operatorname{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsinh(3/2*x - 1/2)
```

mupad [B] time = 0.20, size = 20, normalized size = 1.43

$$\frac{\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 - 6*x + 5)^(1/2),x)
```

```
[Out] log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{9x^2-6x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 - 6*x + 5), x)
```

3.96 $\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ArcSin[1/2 + x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx &= -\left(\frac{1}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -4-8x\right)\right)\\ &= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}+x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] -1/2*ArcSin[(-1 - 2*x)/2]
```

IntegrateAlgebraic [B] time = 0.11, size = 25, normalized size = 2.50

$$-\tan^{-1}\left(\frac{\sqrt{-4x^2-4x+3}}{2x+3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] -ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2*x)]
```

fricas [B] time = 0.40, size = 33, normalized size = 3.30

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-4x^2-4x+3}\,(2x+1)}{4x^2+4x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))
```

giac [A] time = 0.52, size = 6, normalized size = 0.60

$$\frac{1}{2}\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x + 1/2)
```

maple [A] time = 0.05, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-4*x+3)^(1/2),x)
```

```
[Out] 1/2*arcsin(x+1/2)
```

maxima [A] time = 3.01, size = 8, normalized size = 0.80

$$-\frac{1}{2}\,\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
[Out] -1/2*arcsin(-x - 1/2)
```

mupad [B] time = 0.13, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 4*x^2 - 4*x)^(1/2),x)
[Out] asin(x + 1/2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2-4x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-4*x+3)**(1/2),x)
[Out] Integral(1/sqrt(-4*x**2 - 4*x + 3), x)
```

3.97 $\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2} \, dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3
```

IntegrateAlgebraic [A] time = 0.08, size = 24, normalized size = 0.96

$$-\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}-3x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-8 + 6*x + 9*x^2],x]
[Out] -1/3*Log[-1 - 3*x + Sqrt[-8 + 6*x + 9*x^2]]
```

fricas [A] time = 0.41, size = 20, normalized size = 0.80

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

giac [A] time = 0.57, size = 21, normalized size = 0.84

$$-\frac{1}{3}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

maple [A] time = 0.04, size = 30, normalized size = 1.20

$$\frac{\sqrt{9}\ \ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x)
[Out] 1/9*9^(1/2)*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))
```

maxima [A] time = 2.99, size = 22, normalized size = 0.88

$$\frac{1}{3} \log \left(18 \, x + 6 \, \sqrt{9 \, x^{2} + 6 \, x - 8} + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

mupad [B] time = 0.22, size = 20, normalized size = 0.80

$$\frac{\ln\left(3\,x+\sqrt{9\,x^2+6\,x-8}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x + 9*x^2 - 8)^(1/2),x)
```

```
[Out] log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^{2} + 6x - 8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

3.98 $\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx, x, 4+6x\right)}{2\sqrt{6}}\\ &= \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.09, size = 33, normalized size = 1.83

$$-\frac{\log\left(\sqrt{3}\sqrt{3x^2+4x+2}-3x-2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 4*x + 3*x^2],x]
[Out] -(Log[-2 - 3*x + Sqrt[3]*Sqrt[2 + 4*x + 3*x^2]]/Sqrt[3])
```

fricas [B] time = 0.41, size = 38, normalized size = 2.11

$$\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+4x+2}(3x+2)-9x^2-12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5
)
```

giac [B] time = 0.62, size = 33, normalized size = 1.83

$$-\frac{1}{3}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2)
```

maple [A] time = 0.05, size = 15, normalized size = 0.83

$$\frac{\sqrt{3}\ \operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^(1/2),x)
[Out] 1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

maxima [A] time = 2.88, size = 16, normalized size = 0.89

$$\frac{1}{3}\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

mupad [B] time = 0.22, size = 26, normalized size = 1.44

$$\frac{\sqrt{3}\ \ln\left(\sqrt{3}\ \left(x+\frac{2}{3}\right)+\sqrt{3\,x^2+4\,x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 + 2)^(1/2)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**2 + 4*x + 2), x)
```

3.99 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx, x, 4-6x\right)}{2\sqrt{30}}\\ &= -\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

IntegrateAlgebraic [B] time = 0.13, size = 39, normalized size = 2.05

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}-\sqrt{-3x^2+4x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 4*x - 3*x^2],x]
[Out] (-2*ArcTan[(Sqrt[3]*x)/(Sqrt[2] - Sqrt[2 + 4*x - 3*x^2])])/Sqrt[3]
```

fricas [B] time = 0.40, size = 40, normalized size = 2.11

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+4x+2}(3x-2)}{3\left(3x^2-4x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4
*x - 2))
```

giac [A] time = 0.82, size = 16, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2))
```

maple [A] time = 0.05, size = 15, normalized size = 0.79

$$\frac{\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^(1/2),x)
[Out] 1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

maxima [A] time = 2.93, size = 16, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\,\arcsin\left(-\frac{1}{10}\sqrt{10}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

mupad [B] time = 0.14, size = 16, normalized size = 0.84

$$\frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{40}\,(6\,x-4)}{40}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*asin((40^(1/2)*(6*x - 4))/40))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+4x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**2 + 4*x + 2), x)
```

3.100 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
[Out] Log[5 + 6*x + 2*Sqrt[6 + 15*x + 9*x^2]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.12, size = 33, normalized size = 0.94

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3x^2+5x+2}}{\sqrt{3}(x+1)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 5*x + 3*x^2],x]
[Out] (2*ArcTanh[Sqrt[2 + 5*x + 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]
```

fricas [A] time = 0.42, size = 38, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5)+72x^2+120x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
+ 49)
```

giac [A] time = 0.80, size = 34, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

maple [A] time = 0.04, size = 30, normalized size = 0.86

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^(1/2),x)
[Out] 1/3*3^(1/2)*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x+2)^(1/2))
```

maxima [A] time = 3.01, size = 28, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,x^{2}+5\,x+2}+6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

mupad [B] time = 0.24, size = 26, normalized size = 0.74

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{5}{6}\right)+\sqrt{3\,x^{2}+5\,x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^{2}+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**2 + 5*x + 2), x)
```

3.101 $\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx, x, 5-6x\right)}{7\sqrt{3}}\\ &= -\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

IntegrateAlgebraic [B] time = 0.12, size = 35, normalized size = 2.06

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}}{3x+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 5*x - 3*x^2],x]
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[2 + 5*x - 3*x^2])/(1 + 3*x)])/Sqrt[3]
```

fricas [B] time = 0.41, size = 40, normalized size = 2.35

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6\left(3x^2-5x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5
*x - 2))
```

giac [A] time = 0.66, size = 11, normalized size = 0.65

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{6}{7}x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arcsin(6/7*x - 5/7)
```

maple [A] time = 0.05, size = 12, normalized size = 0.71

$$\frac{\sqrt{3}\ \arcsin\left(\frac{6x}{7}-\frac{5}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^(1/2),x)
[Out] 1/3*3^(1/2)*arcsin(6/7*x-5/7)
```

maxima [A] time = 3.00, size = 11, normalized size = 0.65

$$-\frac{1}{3}\sqrt{3}\ \arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

mupad [B] time = 0.17, size = 11, normalized size = 0.65

$$\frac{\sqrt{3}\ \operatorname{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*asin((6*x)/7 - 5/7))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**2 + 5*x + 2), x)
```

3.102 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.81

$$\frac{\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
[Out] Log[2 + 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.09, size = 33, normalized size = 1.03

$$-\frac{\log\left(\sqrt{3}\sqrt{3x^2+4x-2}-3x-2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2 + 4*x + 3*x^2],x]
[Out] -(Log[-2 - 3*x + Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2]]/Sqrt[3])
```

fricas [A] time = 0.41, size = 37, normalized size = 1.16

$$\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+4x-2}(3x+2)+9x^2+12x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

giac [A] time = 0.56, size = 34, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

maple [A] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\sqrt{3}\,\ln\left(\frac{(3x+2)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x-2)^(1/2),x)
[Out] 1/3*3^(1/2)*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^(1/2))
```

maxima [A] time = 2.99, size = 28, normalized size = 0.88

$$\frac{1}{3} \sqrt{3} \log\left(2 \sqrt{3} \sqrt{3 x^2 + 4 x - 2} + 6 x + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

mupad [B] time = 0.23, size = 26, normalized size = 0.81

$$\frac{\sqrt{3}\ \ln\left(\sqrt{3}\ \left(x + \frac{2}{3}\right) + \sqrt{3 x^2 + 4 x - 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 - 2)^(1/2)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 4x - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**2 + 4*x - 2), x)
```

3.103 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 204}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
[Out] -(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])
```

IntegrateAlgebraic [C] time = 0.12, size = 38, normalized size = 1.15

$$\frac{i\log\left(-i\sqrt{3}\sqrt{-3x^2+4x-2}-3x+2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2 + 4*x - 3*x^2],x]
[Out] (I*Log[2 - 3*x - I*Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2]])/Sqrt[3]
```

fricas [C] time = 0.41, size = 65, normalized size = 1.97

$$-\frac{1}{6}i\sqrt{3}\log\left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)+\frac{1}{6}i\sqrt{3}\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
[Out] -1/6*I*sqrt(3)*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) + 1/6*
I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^2 + 4*x - 2), x)
```

maple [A] time = 0.04, size = 26, normalized size = 0.79

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^{2}+4x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x-2)^(1/2),x)
[Out] 1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))
```

maxima [C] time = 2.91, size = 16, normalized size = 0.48

$$-\frac{1}{3}i\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
[Out] -1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

mupad [B] time = 0.14, size = 17, normalized size = 0.52

$$-\frac{\sqrt{3}\ \operatorname{asin}\left(\sqrt{2}\ \left(\frac{3\,x}{2}-1\right)\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 - 2)^(1/2),x)
[Out] -(3^(1/2)*asin(2^(1/2)*((3*x)/2 - 1)*1i))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{2}+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x-2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**2 + 4*x - 2), x)
```

3.104 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] Log[5 + 6*x + 2*Sqrt[-6 + 15*x + 9*x^2]]/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.11, size = 33, normalized size = 0.94

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3x^2+5x-2}}{\sqrt{3}(x+2)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + 5*x + 3*x^2]/(Sqrt[3]*(2 + x))])/Sqrt[3]
```

fricas [A] time = 0.40, size = 38, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5)+72x^2+120x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x
+ 1)
```

giac [A] time = 0.73, size = 34, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

maple [A] time = 0.04, size = 30, normalized size = 0.86

$$\frac{\sqrt{3}\ \ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x-2)^(1/2),x)
[Out] 1/3*3^(1/2)*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x-2)^(1/2))
```

maxima [A] time = 2.93, size = 28, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

mupad [B] time = 0.23, size = 26, normalized size = 0.74

$$\frac{\sqrt{3}\ \ln\left(\sqrt{3}\ \left(x+\frac{5}{6}\right)+\sqrt{3x^2+5x-2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 - 2)^(1/2)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**2 + 5*x - 2), x)
```

3.105 $\int \frac{1}{\sqrt{-2+5x-3x^2}} \, dx$

Optimal. Leaf size=13

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x-3x^2}} \, dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 5-6x\right)}{\sqrt{3}}\\ &= -\frac{\sin^{-1}(5-6x)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

IntegrateAlgebraic [B] time = 0.11, size = 35, normalized size = 2.69

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-3x^2+5x-2}}{3x-2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[-2 + 5*x - 3*x^2])/(-2 + 3*x)])/Sqrt[3]
```

fricas [B] time = 0.41, size = 40, normalized size = 3.08

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x-2}(6x-5)}{6\left(3x^2-5x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5
*x + 2))
```

giac [A] time = 0.64, size = 11, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arcsin(6x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

maple [A] time = 0.04, size = 12, normalized size = 0.92

$$\frac{\sqrt{3}\arcsin(6x-5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsin(6*x-5)
```

maxima [A] time = 3.07, size = 11, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

mupad [B] time = 0.16, size = 11, normalized size = 0.85

$$\frac{\sqrt{3}\,\mathrm{asin}\left(6\,x-5\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin(6*x - 5))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x - 2), x)
```

3.106 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4c}}}\,dx, x, b+2cx\right)}{2c}\\ &= \frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

IntegrateAlgebraic [A] time = 0.26, size = 44, normalized size = 2.00

$$-\frac{\log\left(-\sqrt{c}\sqrt{\frac{b^2}{c}+4bx+4cx^2+4}+b+2cx\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] -(Log[b + 2*c*x - Sqrt[c]*Sqrt[4 + b^2/c + 4*b*x + 4*c*x^2]]/Sqrt[c])
```

fricas [B] time = 0.44, size = 137, normalized size = 6.23

$$\left[\frac{\log\left(-4\,c^2x^2-4\,bcx-b^2-(2\,cx+b)\sqrt{c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}-2\,c\right)}{2\,\sqrt{c}},-\frac{\sqrt{-c}\arctan\left(\frac{(2\,cx+b)\sqrt{-c}\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}}{4\,c^2x^2+4\,bcx+b^2+4\,c}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2 +
 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqrt(
-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2 +
4*c))/c]
```

giac [B] time = 0.75, size = 46, normalized size = 2.09

$$-\frac{\log\left(\left|\left(2\,\sqrt{c}\,x-\sqrt{4\,cx^2+4\,bx+\frac{b^2+4\,c}{c}}\right)\sqrt{c}+b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs((2*sqrt(c)*x - sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c))*sqrt(c) + b)
)/sqrt(c)
```

maple [B] time = 0.08, size = 51, normalized size = 2.32

$$\frac{\sqrt{4}\,\ln\left(\frac{(4cx+2b)\sqrt{4}}{4\sqrt{c}}+\sqrt{4c\,x^{2}+4bx+\frac{b^{2}+4c}{c}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x)
```

```
[Out] 1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*4
^(1/2)/c^(1/2)
```

maxima [A] time = 1.40, size = 16, normalized size = 0.73

$$\frac{\operatorname{arsinh}\left(\frac{2\,cx+b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*(2*c*x + b)/sqrt(c))/sqrt(c)
```

mupad [B] time = 0.42, size = 40, normalized size = 1.82

$$\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,b\,x+4\,c\,x^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2),x)
```

```
[Out] log((b + 2*c*x)/c^(1/2) + ((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2))/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{\frac{b^{2}}{c}+4bx+4cx^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4), x)
```

3.107 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4c}}}\,dx, x, b-2cx\right)}{2c}\\ &= -\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

IntegrateAlgebraic [B] time = 0.34, size = 128, normalized size = 5.57

$$\frac{\sqrt{-c}\log\left(\sqrt{-c}\,cx\sqrt{-\frac{b^2}{c}+4bx-4cx^2+4}-bcx+2c^2x^2-c\right)}{2c}-\frac{\tan^{-1}\left(\frac{2\sqrt{-c}\sqrt{c}x}{b}-\frac{\sqrt{c}\sqrt{-\frac{b^2}{c}+4bx-4cx^2+4}}{b}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcTan[(2*Sqrt[-c]*Sqrt[c]*x)/b - (Sqrt[c]*Sqrt[4 - b^2/c + 4*b*x - 4*c*x
^2])/b]/Sqrt[c]) + (Sqrt[-c]*Log[-c - b*c*x + 2*c^2*x^2 + Sqrt[-c]*c*x*Sqrt
[4 - b^2/c + 4*b*x - 4*c*x^2]])/(2*c)
```

fricas [B] time = 0.46, size = 141, normalized size = 6.13

$$\left[-\frac{\sqrt{-c}\log\left(4\,c^2x^2-4\,bcx+b^2-(2\,cx-b)\sqrt{-c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}-2\,c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}}{4\,c^2x^2-4\,bcx+b^2-4\,c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-(
4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)
)/sqrt(c)]
```

giac [B] time = 1.08, size = 53, normalized size = 2.30

$$-\frac{\log\left(\left|\left(2\sqrt{-c}\,x-\sqrt{-4\,cx^2+4\,bx-\frac{b^2-4\,c}{c}}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c))*sqrt(-c) +
 b))/sqrt(-c)
```

maple [B] time = 0.09, size = 44, normalized size = 1.91

$$\frac{\arctan\left(\frac{2\left(x-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-4c\,x^{2}+4bx-\frac{b^{2}-4c}{c}}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^(1/2))
```

maxima [A] time = 3.08, size = 19, normalized size = 0.83

$$-\frac{\arcsin\left(-\frac{2\,cx-b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/2*(2*c*x - b)/sqrt(c))/sqrt(c)
```

mupad [B] time = 0.41, size = 46, normalized size = 2.00

$$\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{4\,b\,x+\frac{4\,c-b^2}{c}-4\,c\,x^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2),x)
```

```
[Out] log((b - 2*c*x)/(-c)^(1/2) + (4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2))/(-c)^
(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{-\frac{b^{2}}{c}+4bx-4cx^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 4), x)
```

3.108 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c}}}\,dx, x, b-2cx\right)}{c}\\ &= -\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

IntegrateAlgebraic [B] time = 0.33, size = 129, normalized size = 6.45

$$\frac{\sqrt{-c}\log\left(4\sqrt{-c}\,cx\sqrt{-\frac{b^2}{c}+4bx-4cx^2+1}-4bcx+8c^2x^2-c\right)}{2c}-\frac{\tan^{-1}\left(\frac{2\sqrt{-c}\sqrt{c}\,x}{b}-\frac{\sqrt{c}\sqrt{-\frac{b^2}{c}+4bx-4cx^2+1}}{b}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcTan[(2*Sqrt[-c]*Sqrt[c]*x)/b - (Sqrt[c]*Sqrt[1 - b^2/c + 4*b*x - 4*c*x
^2])/b]/Sqrt[c]) + (Sqrt[-c]*Log[-c - 4*b*c*x + 8*c^2*x^2 + 4*Sqrt[-c]*c*x*
Sqrt[1 - b^2/c + 4*b*x - 4*c*x^2]])/(2*c)
```

fricas [B] time = 0.47, size = 143, normalized size = 7.15

$$\left[-\frac{\sqrt{-c}\,\log\left(8\,c^2x^2-8\,bcx+2\,b^2-2\,(2\,cx-b)\sqrt{-c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}-c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}{4\,c^2x^2-4\,bcx+b^2-c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sqr
t(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))/sq
rt(c)]
```

giac [B] time = 0.82, size = 53, normalized size = 2.65

$$-\frac{\log\left(\left|\left(2\sqrt{-c}\,x-\sqrt{-4\,cx^2+4\,bx-\frac{b^2-c}{c}}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c))*sqrt(-c) + b
))/sqrt(-c)
```

maple [B] time = 0.08, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{2\left(x-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-4c\,x^{2}+4bx-\frac{b^{2}-c}{c}}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))
```

maxima [A] time = 2.89, size = 19, normalized size = 0.95

$$-\frac{\arcsin\left(-\frac{2\,cx-b}{\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*c*x - b)/sqrt(c))/sqrt(c)
```

mupad [B] time = 0.40, size = 44, normalized size = 2.20

$$\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{\frac{c-b^2}{c}+4\,b\,x-4\,c\,x^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2),x)
```

```
[Out] log((b - 2*c*x)/(-c)^(1/2) + ((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2))/(-c)^(1
/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{-\frac{b^2}{c}+4bx-4cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 1), x)
```

3.109 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {613}

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + x^2)^(-3/2),x]
```

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(2+3x+x^2\right)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + x^2)^(-3/2),x]
```

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

IntegrateAlgebraic [A] time = 0.18, size = 29, normalized size = 1.53

$$-\frac{2(2x+3)\sqrt{x^2+3x+2}}{(x+1)(x+2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + x^2)^(-3/2),x]
[Out] (-2*(3 + 2*x)*Sqrt[2 + 3*x + x^2])/((1 + x)*(2 + x))
```

fricas [B] time = 0.40, size = 38, normalized size = 2.00

$$-\frac{2\left(2x^2+\sqrt{x^2+3x+2}\,(2x+3)+6x+4\right)}{x^2+3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
[Out] -2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)
```

giac [A] time = 0.83, size = 17, normalized size = 0.89

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")
[Out] -2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

maple [A] time = 0.05, size = 24, normalized size = 1.26

$$-\frac{2(x+2)(x+1)(2x+3)}{\left(x^2+3x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+2)^(3/2),x)
[Out] -2*(x+2)*(x+1)*(2*x+3)/(x^2+3*x+2)^(3/2)
```

maxima [A] time = 1.34, size = 26, normalized size = 1.37

$$-\frac{4x}{\sqrt{x^2+3x+2}}-\frac{6}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

mupad [B] time = 0.05, size = 15, normalized size = 0.79

$$-\frac{4\left(x+\frac{3}{2}\right)}{\sqrt{x^2+3\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + x^2 + 2)^(3/2),x)
```

```
[Out] -(4*(x + 3/2))/(3*x + x^2 + 2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+2)**(3/2),x)
```

```
[Out] Integral((x**2 + 3*x + 2)**(-3/2), x)
```

3.110 $\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {613}

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 37, normalized size = 1.61

$$\frac{(3-x)\sqrt{4x^2-24x+27}}{9(2x-9)(2x-3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] ((3 - x)*Sqrt[27 - 24*x + 4*x^2])/(9*(-9 + 2*x)*(-3 + 2*x))
```

fricas [B] time = 0.40, size = 41, normalized size = 1.78

$$-\frac{4\,x^2+2\,\sqrt{4\,x^2-24\,x+27}\,(x-3)-24\,x+27}{18\,\left(4\,x^2-24\,x+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*x
 + 27)
```

giac [A] time = 0.77, size = 17, normalized size = 0.74

$$-\frac{x-3}{9\,\sqrt{4\,x^2-24\,x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

maple [A] time = 0.05, size = 28, normalized size = 1.22

$$-\frac{(2x-3)\,(2x-9)\,(x-3)}{9\,\left(4x^2-24x+27\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-24*x+27)^(3/2),x)
```

```
[Out] -1/9*(2*x-3)*(2*x-9)*(x-3)/(4*x^2-24*x+27)^(3/2)
```

maxima [A] time = 1.24, size = 30, normalized size = 1.30

$$-\frac{x}{9\sqrt{4x^2-24x+27}}+\frac{1}{3\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.74

$$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 24*x + 27)^(3/2),x)
```

```
[Out] -(x - 3)/(9*(4*x^2 - 24*x + 27)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(4x^2-24x+27\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-24*x+27)**(3/2),x)
```

```
[Out] Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

3.111 $\int \frac{x}{(5-4x-x^2)^{3/2}} \, dx$

Optimal. Leaf size=23

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {636}

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(5-4x-x^2\right)^{3/2}} \, dx = \frac{5-2x}{9\sqrt{5-4x-x^2}}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 1.00

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

IntegrateAlgebraic [A] time = 0.17, size = 33, normalized size = 1.43

$$\frac{(2x-5)\sqrt{-x^2-4x+5}}{9(x-1)(x+5)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(5 - 4*x - x^2)^(3/2),x]
[Out] ((-5 + 2*x)*Sqrt[5 - 4*x - x^2])/(9*(-1 + x)*(5 + x))
```

fricas [A] time = 0.40, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2-4\,x+5}\,(2\,x-5)}{9\,\left(x^2+4\,x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

giac [A] time = 0.61, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2-4\,x+5}\,(2\,x-5)}{9\,\left(x^2+4\,x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

maple [A] time = 0.05, size = 26, normalized size = 1.13

$$\frac{(x+5)\,(x-1)\,(2x-5)}{9\left(-x^2-4x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2-4*x+5)^(3/2),x)
[Out] 1/9*(x+5)*(x-1)*(2*x-5)/(-x^2-4*x+5)^(3/2)
```

maxima [**A**] time = 1.34, size = 30, normalized size = 1.30

$$-\frac{2\,x}{9\,\sqrt{-x^2-4\,x+5}}+\frac{5}{9\,\sqrt{-x^2-4\,x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")
[Out] -2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

mupad [**B**] time = 0.05, size = 19, normalized size = 0.83

$$-\frac{2\,x-5}{9\,\sqrt{-x^2-4\,x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5 - x^2 - 4*x)^(3/2),x)
[Out] -(2*x - 5)/(9*(5 - x^2 - 4*x)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(-\left(x-1\right)\left(x+5\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2-4*x+5)**(3/2),x)
[Out] Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

3.112 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2}{27}\int \frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx\\ &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
[Out] -1/243*((2 + x)*(-19 + 8*x + 2*x^2))/(5 - 4*x - x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.28, size = 43, normalized size = 1.00

$$\frac{\sqrt{-x^2-4x+5}\left(-2x^3-12x^2+3x+38\right)}{243(x-1)^2(x+5)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(5 - 4*x - x^2)^(-5/2),x]
[Out] (Sqrt[5 - 4*x - x^2]*(38 + 3*x - 12*x^2 - 2*x^3))/(243*(-1 + x)^2*(5 + x)^2
)
```

fricas [A] time = 0.43, size = 49, normalized size = 1.14

$$-\frac{\left(2\,x^3+12\,x^2-3\,x-38\right)\sqrt{-x^2-4\,x+5}}{243\left(x^4+8\,x^3+6\,x^2-40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

giac [A] time = 0.78, size = 36, normalized size = 0.84

$$-\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\left(x^2+4\,x-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

maple [A] time = 0.04, size = 36, normalized size = 0.84

$$\frac{(x+5)(x-1)\left(2x^3+12x^2-3x-38\right)}{243\left(-x^2-4x+5\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(x+5)*(x-1)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

maxima [A] time = 1.41, size = 59, normalized size = 1.37

$$\frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}}+\frac{4}{243\,\sqrt{-x^2-4\,x+5}}+\frac{x}{27\left(-x^2-4\,x+5\right)^{\frac{3}{2}}}+\frac{2}{27\left(-x^2-4\,x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

mupad [B] time = 0.03, size = 29, normalized size = 0.67

$$-\frac{(4\,x+8)\,\left(8\,x^2+32\,x-76\right)}{3888\left(-x^2-4\,x+5\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5 - x^2 - 4*x)^(5/2),x)
```

```
[Out] -((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^2-4x+5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

3.113 $\int (3+4x)^p \, dx$

Optimal. Leaf size=18

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)^p,x]
```

```
[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (3+4x)^p \, dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{(4x+3)^{p+1}}{4p+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)^p,x]
```

```
[Out] (3 + 4*x)^(1 + p)/(4 + 4*p)
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (3+4x)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*x)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(3 + 4*x)^p, x]
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{(4x+3)^p(4x+3)}{4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

giac [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

maple [A] time = 0.04, size = 17, normalized size = 0.94

$$\frac{(4x+3)^{p+1}}{4p+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)^p,x)
```

```
[Out] 1/4*(3+4*x)^(p+1)/(p+1)
```

maxima [A] time = 1.25, size = 16, normalized size = 0.89

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

mupad [B] time = 0.39, size = 32, normalized size = 1.78

$$\begin{cases} \frac{\ln(4\,x+3)}{4} & \text{if } p=-1 \\ \frac{(4\,x+3)^{p+1}}{4\,(p+1)} & \text{if } p\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)^p,x)
[Out] piecewise(p == -1, log(4*x + 3)/4, p ~= -1, (4*x + 3)^(p + 1)/(4*(p + 1)))
```

sympy [A] time = 0.06, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(4x+3)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(4x+3) & \text{otherwise} \end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)**p,x)
[Out] Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4
```

Chapter 4

Appendix

Local contents

4.1 Download section 458
4.2 Listing of Grading functions 458

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)



GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
   gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
   Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
   Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
   Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
   [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
   isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```